



Edic Baghdasarian

A History of Mathematics in Armenia



Anania Shirakatsi, 7th century

**By
Edic Baghdasarian**

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Table of Contents

Foreword	7
Section 1	9
An overview of the history of mathematics	9
Section 2	14
A brief HISTORY OF ARMENIA.....	14
Map of Armenia (through the centuries)	14
Ancient Armenia (3500 BC - 520 BC).....	14
Artashesian dynasty.....	16
Arshakunian dynasty	18
Arab invasion and Byzantine Empire.....	19
Bagratunian dynasty	20
The Armenian Kingdom of Cilicia.....	21
The Armenian Kingdom of Cilicia.....	21
Armenia under Turkish, Persian and Russian rule	22
Armenian Genocide.....	24
Republic of Armenia	25
Section 3	27
Mathematics during Araratian.....	27
or Van kingdom (Known as Urartu).....	27
Section 4.....	40
Armenian Mathematical Sciences after.....	40
Urartu until the seventh century AD	40
Section 5.....	50

Anania Shirakatsi, a great scientist of the 7th century AD and the importance of his book “Arithmetic”	50
Status of arithmetic prior to the seventh century AD	59
Section 6	81
Leon, the famous mathematician of the ninth century AD.	81
Section 7	84
Grigor Magistros,	84
translator of the oldest Armenian version of	84
Euclid’s geometry	84
Section 8	93
Hovhannes Sarkavak and "Polygonal numbers"	93
Section 9	107
Mathematical works of	107
Nikoghayos (Nokolay) Artavazd	107
Section 10	116
Application of decimal numbers in Armenia	116
based on the Armenian sources	116
Section 11	134
Armenian mathematical resources	134
(17-18 centuries)	134
Section 12	164
Armenian mathematics	164
in the 19th and 20th centuries	164
Section 13	172
Prominent Armenian mathematicians	172
in Armenia and Diaspora	172
Appendices	191
Appendix 1	193
Mathematical problems of Anania Shirakatsi	193
Appendix 2	199
Mathematical Entertainment of Anania Shirakatsi	199

Appendix 3	201
Polygonal numbers	201
of.....	201
Hovhannes Sarkavak	201
Appendix 4	206
Examples of the mathematical problems.....	206
by Nikoghayos (Nikolai) Artavazd	206
Appendix 5	209
A glossary of mathematical terms	209
Index	219
Bibliography	226

Foreword

The idea of writing a book on the history of the Mathematics in Armenia came to my mind many years ago, followed by several papers that I presented to the International Seminars on the History of Mathematics. However, my invitation to these seminars grabbed the attention of participants, mathematicians and scientists towards the history of the Armenians in science and scientific works, so I was proposed by them and Iranian Ministry of higher Education, Scientific Research Organization to compile a book about this subject.

The main sources for writing this book were Works of Professor G. B. Petrosyan, a four-volume "History of natural sciences and technology among Armenians", "Armenian cultural history of ideas", resources on mathematic at the National Library manuscripts known as Mesrop Mashtots Matenadaran in Yerevan, capital of the Republic of Armenia, eight-volume History of Armenia, Armenian culture, Iranian resources, many websites, and so on.

I would like to dedicate this book to all scientists, from any nation or ethnicity, race or religion especially the mathematicians, hoping that the science and the scientists will have their own adequate position, respect and appreciation.

I would like to thank my dear scientist friends, seminar participants and Ministry officials who encouraged me to accomplish this work, also my great thanks to Dr. M. Toumanian for his useful comments and guidance and other feedbacks. I hope this book will be helpful for all readers and especially the researchers and those who are interested in the history of science.

This work was originally in Persian and printed in 2000 in Tehran and second time in 2007 in Ottawa. I had many suggestions for the English version, which I accepted and now it is printed.

I would appreciate any comments and suggestions to my potential errors for them I need to apologize and will do my best to improve the quality of this research for next printings.

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August 6, 2015

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Section 1

An overview of the history of mathematics

The development of mathematics in the course of history has passed through various phases, which is presented briefly in this part of the book:

1. Ancient era (from ancient times until 5-6 centuries BC)

During this period, over many centuries, based on the number of animals, humans and objects, the concept of natural numbers appeared at the beginning, then the need to count the grains and measure the length of roads and the area of lands created naming of simple fractional values and a method for writing them and eventually documented rules and methodologies. Thus arithmetic gradually emerged. At later time the need to measure the areas and technical prerequisites for building and construction caused the emergence of one of the ancient branches of the mathematical sciences i.e. geometry. Arithmetic and geometry appeared independently among many nations almost in the same time and in parallel.

The oldest mathematical resources are Egyptian mathematical papyrus (second millennium BC) and the Babylonian mathematical cuneiform texts (the second and first millennium BC), respectively. According to the needs of astronomy, trigonometry was created. Creation of mathematical cuneiform in Assyrian, Persian, Urartu and Hellenistic world continued until the first century BC. The mathematics was not presented in an organized way, most of arithmetic and geometrical collection of problems were unrelated to each other and unique rules and principles of problem

solving techniques were missing, so the only way to find out the methodology was examining the problem solution.

2. The second stage or era begins from the sixth century BC. During this era the mathematics gradually transformed into a structured knowledge associated with reasonable and logical concepts. In this way the Greeks have taken the first steps. Arithmetic in the school of Pythagoras was converted from a simple counting into numbers theory.

Mathematics, in the third century BC, had great progress and development in the city of Alexandria, the scientific center of the Hellenistic world. Euclid collected all the rules and the results of past experiences and presented the structured theory of numbers. Archimedes calculated approximate areas. He calculated weighted centers and proved that number Pi was between $(3 \text{ and } 10/71)$ and $(3 \text{ and } 10/70)$. Later on the geometry of the counting (Heron, the first century) and spherical trigonometry (Ptolemy, the second century) was set up and the rules of first and second degree equations were developed (Diophantus, the third century) and for the first time the concept of negative numbers was introduced. From the third century onwards the culture of the ancient world, in general, declined and as a result mathematics was declined and Greek mathematicians were mainly busy with interpretation of the works of earlier authors.

In the 15-9 centuries, in the East, Arabs translated the Greek and Indian mathematical works to their own language, and the Europeans in a later time got familiar with the works of ancient Greek and Indian mathematics through the same sources.

In this period, great scientists appeared in the world of mathematics such as Al-Khwarizmi (9th century), Biruni (10-9), Abu al-Wafa Buzjani (10), Omar Khayyam (12-11M.), Nasir al-Din Tusi (13 m.) and others. European “algorithm” is derived from the name al-Khwarizmi. By modern

science of algebra, Omar Khayyam besides Algebraic Methods wrote interpretations of the "principles" of Euclid and founded trigonometry.

Until the 17th century mathematical studies, except for exceptional cases, consisted of fixed numbers and the relationship between them, so from ancient times to the 17th century it is considered the era of mathematics of fixed values.

3- Starting the 17th century, new era of mathematics called mathematical reformation began with the mathematics of variables. New socio-economic relations, the rapid development of ocean shipping, shipbuilding, production raised up new needs that math of constants and method of Archimedean mechanics were not able to solve them. Natural sciences entered a new phase of development; falling objects (Galileo), the motion of planets (Kepler), gravity (Newton), etc. were discovered. The important thing is that the equations and mathematical formulas were defined as the laws of the various phenomena and as a result, natural scientific mathematics was established, which in its turn, caused new achievements in mathematics. By the generalization of the discussion about the time, distance, direction, speed and other physical concepts, abstract concepts in mathematics was put on the table and generalized reciprocity variable physical quantities led to the concepts of the functions.

This also happened in mathematics and Scottish mathematician and physicist John Napier (1550-1617) defined logarithm. Rene Descartes (1637) defined coordinates method and used algebra in geometry and expressed the lines through equations. Fermat (1638) was able to find the maximum and minimum of the functions. During the years 1630-60 different solutions to various problems in geometry and physics were presented. In 1665-84 general concepts of variables and integral and differential equations were created and the solutions were presented, thus

Calculus of integrals and Differentials was emerged as a new science of mathematics. A group of scientists such as Leibniz, Newton, Bernoulli, Hopital, Euler, Lagrange, Laplace, Legendre, d'Alembert, February, Taylor, Gauss and others were involved in this development.

4 - Since the first half of the 19th century, begins the new era of today's mathematics. Major characteristics of this period are the revision and modernization of the basic mathematical concepts and definition of all operations related to them based on logic. Accordingly, a new look at natural phenomena based on mathematical equations were generalized and allocated. Instead, vague concepts in the past were replaced by clear and definite values (extremely small values, functions and scope of the concept of continuity, the theory of real and complex numbers, proving many of the theories, the theory of limits, the basic formulation of calculus of integrals and differentials etc.). Thus the analysis of extremely small numbers arose which later on became known as mathematical analysis. In this regard, as a result of independent researches, the following fields of mathematical analysis such as the theory of functions of real variables, the theory of functions of complex variables, differential equations, calculus of variations, analysis of functions, etc. appeared which are widely used in other parts of mathematics and other sciences.

The emergence of non-Euclidean geometry of Lobachevsky created a great revolution in the field of geometry. B. Riemann ideas had great influence upon subsequent developments of the geometric theory of functions of complex variables. D. Hilbert formulated the principles of geometry and geometry of the principles was created. In the twentieth century differential geometry and topology (local analysis, topology) were created.

Algebra was transformed as a result of the many studies and large-scale generalized algebraic methods and the Theory of sets, mathematical

structures have influences in all areas and have found wide applications in many fields of science.

Probability Theory equipped with variables functions, differential equations, Theoretical methods of size were equipped with rapid progress have applications in many fields of natural sciences, economic and military. New fields such as mathematical statistics, theory of stochastic processes, information theory, game theory, etc. were created.

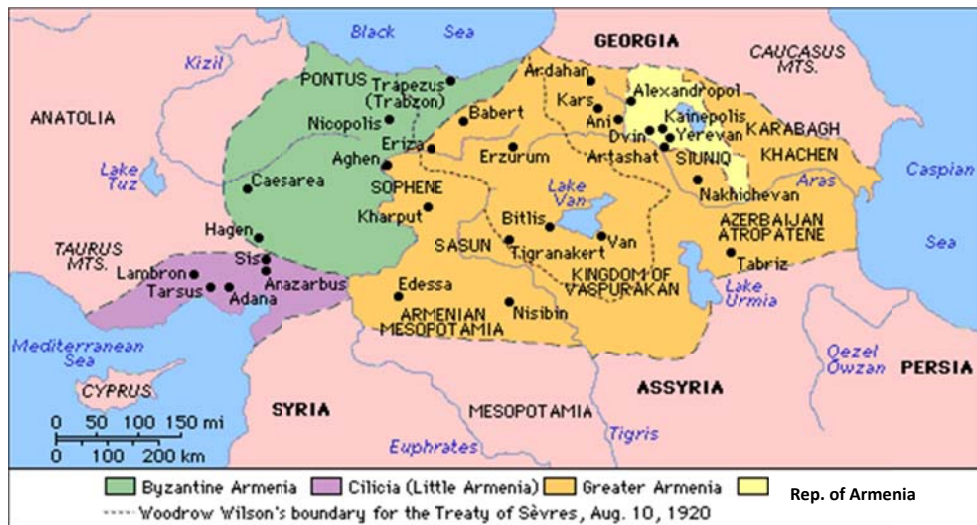
The new and modern mathematical achievement and innovation based on quantitative and qualitative expansion of relations are progressing with the predefinitions about the internal needs of mathematics. General and abstract ideas are formed and new proof of the theories underlying their logic is necessary and unique problems in mathematics, mathematical logic emerge.

The practical application of the theoretical studies of mathematics requires a numerical response to the proposed problems. Numerical analysis techniques developed in the late nineteenth century and extensive tables of mathematical calculations were made in order to facilitate and accelerate them. From the middle of the twentieth century, mathematical cybernetics gets promoted and electronic calculators or computers got built rapidly and have been widely applied in all areas of human life, and mathematical methods penetrate unprecedentedly in all scientific, economic and industrial fields.

Today, with globalization of computer applications, especially with the emergence of the World Wide Web, mathematics as the mother of sciences has its undeniable role in all aspects of the present and future human life.

Section 2

A brief HISTORY OF ARMENIA



Map of Armenia (through the centuries)

Ancient Armenia (3500 BC - 520 BC)

Armenia is one of the oldest countries in the world with a recorded history of about 3500 years. The oldest known ancestors of modern Armenians, the Hayasa-Azzi tribes, also known as Proto-Armenians, were indigenous to the Armenian Highland in Eastern Anatolia. These tribes formed the Nairi tribal union, which existed until late 13th century BC. The legendary forefather of Armenians, Hayk, famous for his battles with Babylonian ruler Bel, most likely was one of the Hayasa tribal leaders. The words *Nairi* and *Nairian* are still used by Armenians as poetic synonyms of the words *Armenia* and *Armenian*.

At the end of the second millennium BC, another Indo-European ethnic group, closely related to Thracians and Phrygians and referred to Greeks as Armens, migrated to the Armenian Highland from Northern Balkans. According to a Greek myth, which actually reflects this tribal migration, the forefather of Armenians - Armenios - was one of the Argonauts, accompanying Jason in his quest for the Golden Fleece. In the year 1115 BC, King Tiglath Pileser I of Assyria reports a battle with a force of 20,000 Armens in the Gadmokh province of Assyria.

The Kingdom of Urartu, 9th-6th Centuries B.C.



The mixture of Armens with the indigenous Hayasa eventually produced the Armenian people as it is known today. The existence of two major segments in the Armenian people is best of all illustrated by the fact that Armenians call themselves "Hay" and their country "Hayastan" after Hayasa, while other peoples call them Armenians and their country Armenia after the Armens. The Armenian language is basically the language of Armens. It incorporated a large number of Hayasa words and

grammatical features, as well as a significant number of non-Indo-European words from minor ethnic groups, which also took part in the ethnogenesis of Armenians.

The first significant state of the Armenian Highland was the highly advanced Kingdom of Ararat (with the capital in Tushpa, today's Van), better known under its Assyrian name Urartu (Ararat). This state was formed in the XI century BC and existed until VII century BC. In 782 BC the Urartian king Argishti I founded the fortified city of Erebuni, which is today's Yerevan, the capital of the Republic of Armenia. Another major city in the Valley of Ararat was Argishti-khinili, also founded by Argishti I in the year 775 BC.

In the late VII century BC Urartu, weakened by Scythian invasions fell, but after several decades was revived under the Armenian Yervanduni (the Orontides) dynasty with the capital in Armavir, former Argishti-khinili. The revived kingdom was already called Armenia by its neighbours, but in some languages the older name, Urartu, was still in use. In the famous trilingual Behistun inscription of Persian king Darius the Great (522-486) the same country is referred to as *Armenia* in the Persian and Elamite versions and *Urartu* in the Akkadian version.

Artashesian dynasty

Armenia under the Yervanduni dynasty soon became a satrapy of the mighty Achemenide Persia, and later part of the Seleucid Empire. It restored its full independence in 189 BC under the king Artashes I, founder of the Artashesian dynasty (the Artaxiads).



The kingdom started to expand and reached its peak during the reign of Tigran II, also called Tigran the Great (95-55 BC). Under Tigran, Armenia ascended to a pinnacle of power unique in its history and became the strongest state in Asia Minor. Extensive territories were taken from Parthia, which was compelled to sign a treaty of alliance. Iberia (Georgia), Caucasian Albania, and Atropatene had already accepted Tigran's sovereignty when the Syrians offered him their crown (83 BC). Tigran penetrated as far south as Ptolemais (modern Akko in Israel). As a result, the empire of Tigran II stretched from the Caspian Sea in the East to the Mediterranean Sea in West, and from Mesopotamia in the South to the river Kura in North. Political strengthening and territorial expansion of Armenia was accompanied also by unprecedented cultural development, with rich cultural heritage of Urartu intermixing with Hellenistic features. As a result Armenia during the Artashesian period became one of the most Hellenized and culturally advanced countries of Asia Minor.

After the death of Tigran II, Armenia was reduced back to its ethnic Armenian territory and found itself in the middle of a long war campaign between Rome and Persia, with each superpower trying to have Armenia

as its ally, as the military assistance with Armenia was crucial for gaining political superiority in Asia Minor.

Arshakunian dynasty



In the middle of the first century AD a new royal dynasty - the Arshakuni or Arshakunian (the Arsacids) - was established in Armenia. This dynasty was related to the royal family of Persia, which bared the same family name. At this period Armenia and Persia enjoyed a long period of peace and cooperation, until in 224 AD the Sassanid dynasty came to power in Persia. Regarding Armenia as the ally of the overthrown dynasty, the Sassanids adopted anti-Armenian policy, trying to eliminate the Armenian state and to assimilate the Armenian nation. Since the Armenian religion of that period bared similarities to both Zoroastrianism and Greco-Roman polytheism, in the realization of their anti-Armenian policy the Sassanids were trying to capitalize on the religious closeness. In order to deprive the Persians of this advantage, the Armenian king Trdat III in 301 AD

declared Christianity the state religion of Armenia, thus making Armenia the first Christian state in the world, with Gregory the Illuminator as the first head (Catholicos) of the Armenian Apostolic Church. Christianity was officially legalized in the Roman Empire 12 years after Armenia became officially Christian.

Arab invasion and Byzantine Empire

By the end of the 4th century the Byzantine Empire and Sassanid Persia officially established their spheres of influence in Armenia. The Arshakuni dynasty was dissolved in the year 428, and eastern part of Armenia was annexed to Persia, while the western part was put under Byzantine rule. The Sassanids were forcing Armenians to convert to Zoroastrianism, causing the Armenian revolt of 451 under the leadership of Prince Vartan Mamikonian, commander-in-chief of the Armenian army. Although the Armenian forces, outnumbered by the Persians, actually lost the legendary battle of Avarayr, and Vartan Mamikonian himself was killed, it turned out to be a significant victory for Armenians, as Persians eventually gave up their efforts to convert and assimilate Armenians, and were forced to agree with much higher level of autonomy for Armenia.

The spiritual independence of Armenia was further asserted in 554, when the second Council of Dvin (capital of Armenia of that period) rejected the dyophysite formula of the Council of Chalcedon (451), a decisive step that cut Armenians off from the Roman and Greek churches as surely as they were already ideologically severed from the East.

By the time of Arab invasion in 634, Armenia ruled by Prince Theodore Rshtuni, was virtually independent. After conquering Persia, the Arabs started to concentrate their armies against Armenia, but didn't manage to conquer the country until 654.

Bagratunian dynasty



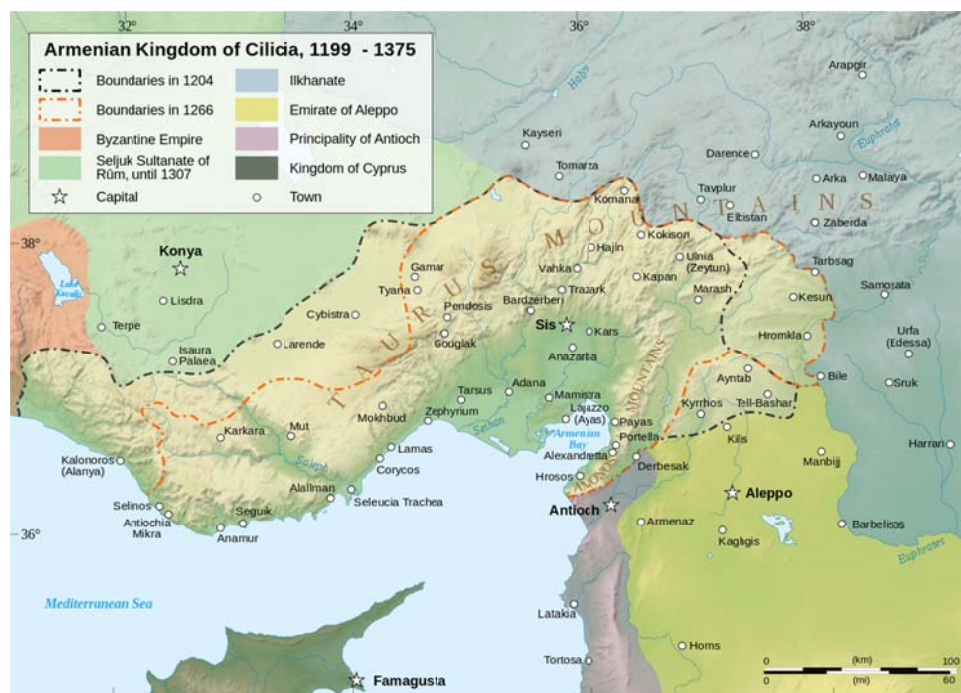
After more than two centuries of struggle with the Arab Caliphate, Armenia regained its independence in 886, and both the Caliphate and Constantinople recognized Prince Ashot Bagratuni as the king of Armenia. During the rule of the Bagratuni dynasty Armenia reached its peak in political, social and cultural development. The capital of Armenia of that period, Ani, was a magnificent city, known as "a city of one thousand and one churches". The Armenian architecture of the Bagratuni period, especially the dome laying techniques, for which Armenian architects were famous, significantly influenced the Byzantine and European architectural styles.

At the end of the 10th century the Byzantine Empire, although ruled by an imperial dynasty of Armenian origin, adopted a near-sighted policy of weakening Armenia and eventually annexed it in 1045, thus depriving

itself of an effective shield against disastrous invasion of Turkic nomads from Central Asia.

The Armenian Kingdom of Cilicia

Before the fall of the Bagratuni kingdom a number of Armenian princes managed (or were forced by Byzantines) to escape from Armenia and found refuge in Cilicia, a region at the north-eastern corner of the Mediterranean Sea, where Armenians were the majority of population.



The Armenian Kingdom of Cilicia

In 1080 their leader, Prince Ruben, founded in Cilicia a new kingdom, which became known as Armenian Kingdom of Cilicia. The new Armenian state established very close relations with European countries

and played a very important role during the Crusades, providing the Christian armies a safe haven and provision on their way towards Jerusalem. Intermarriage with European crusading families was common, and European religious, political, and cultural influence was strong. The royal court of Cilicia and the kingdom itself were reformed on Western models, and many French terms entered the Armenian language. Armenian Cilicia also played an important role in the trade of the Venetians and Genoese with the East.

Enduring constant attacks by the Turks, Mongols, Egyptians and Byzantines, Cilician Armenia survived for three centuries and fell to Egyptian Mameluks in 1375, but survived partially until 1424. The last Armenian king of Cilicia, Levon VI Lousinian emigrated to France, where his grave still can be seen in the St. Denis Cathedral of Paris. The title "King of Armenia" passed to the kings of Cyprus, thence to the Venetians, and was later claimed by the house of Savoy (a cultural region in Rhône-Alpes, France).

Armenia under Turkish, Persian and Russian rule

After the fall of the Armenian kingdom in Cilicia, the historical Armenian homeland was subject to various Muslim warlords, and eventually was divided between the Ottoman Empire (Western Armenia) and Persia (Eastern Armenia). Several Armenian principalities managed to preserve their independence or autonomy. The most significant among those was the Federation of Khamsa in Artsakh (today's Nagorno-Karabakh), which consisted of five allied principalities. De facto independent Armenian principalities existed also in the regions of Sasun and Zeytun in Western Armenia.

Being for centuries at the edge of physical annihilation, Armenians nevertheless managed to preserve and develop their national, religious and cultural identity. Apart from architecture, Armenians successfully

manifested themselves in literature, painting, sculpture and music. Armenians were the 11th nation in the world to put their language in print (since 1512).

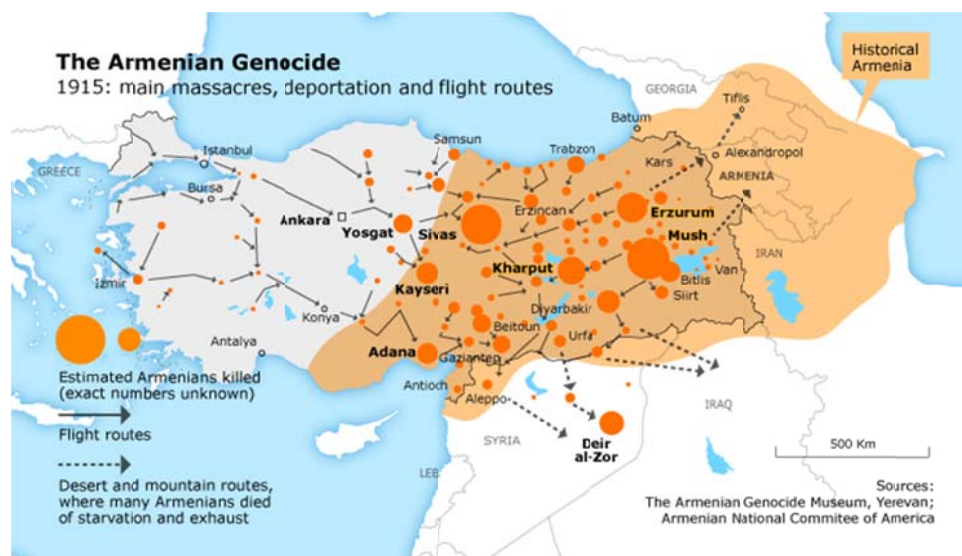
In 1828 the Russian Empire captured Eastern Armenia from Persia. Contact with liberal thought in Russia and Western Europe was a factor in the Armenian cultural renaissance of the 19th century. In the Ottoman Empire, the Armenians initially benefited with the rest of the population from the measures of reform known as the Tanzimat, and in 1863 a special Armenian constitution was recognized by the Ottoman government. These liberties were however unknown outside Constantinople, and the condition of Armenians in Anatolia was unbearable. After the Russo-Turkish War of 1877-78, in which Eastern Armenians had taken part, Russia insisted in the Treaty of San Stefano that reforms be carried out among the sultan's Armenian subjects and that their protection against the Kurds be guaranteed. This demand was softened at the Congress of Berlin.



Western and Eastern Armenia under Ottoman and Russian rule

Having lost most of its territory in the Balkans, the Ottoman Empire was afraid of losing Western Armenia as well, which would mean the end of the Ottoman dream of creating a pan-Turkic empire, stretching from Balkans to the Yellow Sea. A new state policy was formed, aiming at the final resolution of the Armenian Question through total annihilation of Armenians in their historic homeland of 3500 years. During the reign of sultan Abdulhamid Armenian massacres became a common phenomenon. In 1895, after Abdulhamid had felt compelled to promise Britain, France, and Russia that he would carry out reforms, large-scale systematic massacres took place in the Armenian provinces. In 1896 more massacres broke out in the capital and in Cilicia.

Armenian Genocide



After coming to power in Constantinople, the Young Turks made the policy of "No Armenians - no Armenian Question" their main priority. Taking advantage of the favourable political conditions created by the World War I, they began the "final resolution of the Armenian question"

on April 24th, 1915, by executing hundreds of Armenian intellectuals of Constantinople without trial. In Armenian provinces of Eastern Anatolia all Armenian males aged 15-62 were conscripted, disarmed and executed. Defenceless Armenian women, children and the elderly were deported to the Syrian desert of Der-el-Zor; most of them were brutally murdered on the way by Turkish soldiers or Kurdish nomads, or died of starvation and exhaustion. More than one and half million Armenians, i.e. 80% of the Armenian population of Western Armenia, perished in this first Genocide of the twentieth century. Several hundred thousand survivors of the Genocide found refuge in neighbouring counties, laying the foundation of the worldwide Armenian Dispersion or Diaspora. By the year 1923 Western Armenia was completely de-Armenized, and successfully incorporated into the newly formed Turkish Republic.

Republic of Armenia

Between 1915 and 1917, Russia occupied virtually the entire Armenian part of the Ottoman Empire. Then in October 1917, the Bolshevik victory in Russia ended that country's involvement in World War I, and Russian troops left the Caucasus. In the vacuum that remained, the Armenians first joined a Transcaucasian federation with Azerbaijan and Georgia, both of which, however, soon proved to be unreliable partners. The danger posed by the territorial ambitions of the Ottoman Turks and the Azerbaijanis finally united the Caucasian Armenian population. In May 28, 1918, an independent Armenian republic was declared; its armies continued to fight on the Allied side south of the Caucasus until the Ottoman Empire surrendered in October 1918. The independent republic endured from May 28 1918 to November 29 1920.

In August 1920, the Treaty of Sevres, signed by England, France and Turkey, bound Turkey to recognize the independence of Armenia and the Wilsonian boundaries. The new Armenian state was recognized by most of the countries, including the United States. However, after the triumph of Mustafa Kemal, the Turks, supported by the Bolshevik Russia, attacked

the infant Armenian Republic again. The victorious Russian XI Red Army, after successfully Sovietizing Baku, Azerbaijan, and Karabakh, approached Yerevan. On November 29, 1920, Armenia was declared a Soviet state.



Armenia restored its full independence on September 21, 1991, and became a member of the United Nations on March 2, 1992. On January 25, 2001, Armenia also became a member of the Council of Europe.¹

¹ - United Nations (Official Site)

- Great Armenian Encyclopedia, Vo. 6, p.158, Yerevan, 1980.
- Movses Khorenatsi, "History of the Armenians", Yerevan, 1968, p.76.
- Ferdowsi, "Shahnameh", Tehran, 1998, pp 42-44.
- B. Farahvashi, Dictionary of Palavi language, Tehran, 1979.
- S. Bahrami, A Dictionary of Avesta, 4 vol., Tehran 1980.

Section 3

Mathematics during Araratian or Van kingdom (Known as Urartu)

In ancient times, numbers and simple fractions formed the basis of arithmetic. Arithmetic, as part of mathematics investigated the numbers (natural numbers and fractions in the first place) and operations related to them.

The Man before writing the words, tried the numbers. According to the method of writing numbers we can judge about the level of civilization and culture in various geographical areas and communities.

Mathematics was established when the man in the different areas of the earth in the course of his struggle for life tried to find out the basis of the numbers and during the time improved it to scientific level.

Man for a variety of measurements used his body organs as a unit, thus applied his fingers and toes for calculations. That is why the numbers 5, 10, 20 are the basis of counting.

Counting system based on 12 was mostly used by the Romans. But today, all the nations of are using mathematical signs and numbers known as Indian and Arabic numbers.

Surprisingly humanity only in using similar numeric systems has achieved high level of communal and mutual life and similarity and solidarity. However, in other areas, different people have different opinions and deep disagreements.



Caucasus rulers clay tablets addressed a king of Urartu.
Tupraq- castle. Before the end of the seventh century BC

Castles, palaces, cathedrals and churches, bridges and irrigation systems and buildings which were built several millennia ago in Armenia and other Armenian-populated areas which still remain, are evidence to high level of mathematical knowledge among the Armenians. Armenians before Christ and especially during Araratian state (mistakenly widely known as Urartu) had specific signs and calculation methods.

This section examines the mathematical methods and applications on tablets and cuneiform inscriptions of Urartu during the 9 to 7 centuries BC.

About Urartu (9 to 7 centuries BC)

The oldest sources of information about the land and the state of Urartu are Assyrian inscriptions in cuneiform texts. The land in ancient Hebrew is called as the land of Ararat, in ancient Armenia there was a land called Ayrarat from which the name of Mount Ararat was derived. The center of this land was Biayna which lateron was transformed to Van. The first parts

of the names Aria, Armenia, Ararat, Ayrarat², Urartu, Iran and other names, have similar meaning which is “pure race” and refers to great Aryan or Indo-European race so it seems that in order to study the culture and civilization of all Arian populated regions, we need to focus on the areas having this mutual particle of their names, thus we try to present a brief report on the calculation and counting methods in this region.

So far, no archive on the land at the time has been reached. About the Urartu culture and science in general, we should refer to the remains of material culture, especially cuneiform inscriptions.

According to reliable historical sources, such as the history of Moses Khorenatsi and Assyrian sources, numerous references are made to techniques of construction and development projects in the territory of Urartu. Khorenatsi in his history (fifth century AD) wrote: "shamiram ordered construction of a wide and high dam with large stones and lime mortar and sand over the river, which remains until our times ... So she built a long canal that stretched from the reservoir to the city. She also built two and three-storey buildings inside the city. Alleys and streets of the city were built in a beautiful and appropriate width. Also some bathrooms were built and were suitable for the needs of the building³". The creek or canal has a length of 70 kilometers, which was actually constructed by Menua king of Urartu⁴. There are other examples of such development operations.

Thus, we must conclude that in order to create canals, dams, buildings and the town, mathematics has played a key role, so computational principles must have been available at the level of sciences of the time.

² - B.B. Piotrovsky, “Urartu”, Translated by E. Reza, Tehran, 1969, p. 15.

³ - Movses Khorenatsi, “History...”, Yerevan, 1940, Book 1, chapter 16.

⁴ - Prof. G. Ghapantsian, “History of Urartu”, Yerevan, 1940.

Calculation system

Reliable sources about the history of mathematical and computational techniques in Urartu are inscriptions and petrographs. The people of this land in order to meet the increasing needs in the field of computational techniques and writing, in the ninth century BC used hieroglyphic instead of cuneiform tablets (borrowed from the Assyrian Babel).



Cuneiform inscription by Sarduru king of Urartu, mid 8th century BC

The cuneiform underwent some changes in Urartu. Babylonians wrote on clay boards and letters linearly with triangular ending. Since Urartu inscriptions were carved on the flat and trimmed stones, so their letters were cuneiform scripts and had triangular stretched ending. We have more

than 300 inscriptions in hand so far. We used those inscriptions that belong to the times of Sarduri the Second, which have been discovered by famous Armenian scientist Hovsep Orbeli during 1915 in the city of Van⁵.

In fact, the majority of conclusions about mathematics at that time is obtained from inscriptions of Sarduri the Second.

Sarduri the Second, in the first tables of the inscription mentions how the God Khald made him win. He continued the battle from three directions and captured some captives and spoils. In his inscriptions, he describes that: "Sarduri the Second during his first expedition to the East (Babylon, etc.) captured 23 cities and 8135 children, 25,000 women and 6,000 men, and 2,500 horses, 12,300 cattle and 32,100 sheep were taken as spoils". Numbers are as follows:

23—⟨⟨ΥΥ
 8 135—ΥΥΥΥ ⟨Υ- Υ Υ- <<< ΥΥ
 25 000—⟨⟨ ⟨Υ- ΥΥ ⟨Υ-
 6 000—ΥΥΥ ⟨Υ-
 2 500—ΥΥ ⟨Υ- ΥΥ Υ-
 12 300—⟨ ⟨Υ- ΥΥ ⟨Υ- ΥΥ Υ-
 32 100—<<< ⟨Υ- ΥΥ ⟨Υ- Υ Υ-

The first expedition Statistics

In the second attack in the north 3,500 children, 10,500 women and 4,000 men were captured and 8,525 cattle and 18,000 sheep were taken. Numbers are as follows:

⁵ - H. Orbeli, N. Marr, "Archeological researched in Van, Petersburg, 1922.

3 500—𐎶𐎶𐎶 𐎠𐎵 𐎶𐎶𐎶 𐎶𐎵
10 500 ————— 𐎠𐎵 𐎶𐎶𐎶 𐎶𐎵𐎶
4 000—𐎶𐎶 𐎠𐎵
8 525—𐎶𐎶𐎶𐎶 𐎠𐎵 𐎶𐎶𐎶 𐎶𐎵 𐎠𐎶𐎶𐎶
18 000—𐎠𐎵 𐎠𐎵 𐎶𐎶𐎶𐎶 𐎠𐎵

The second expedition statistics

In his third expedition to East (Urmia, etc.) 1,100 children, 6,500 women, 2,000 men and 2,538 cattle and 8,000 sheep were captured. In the following lines numbers used in the cuneiform inscriptions are mentioned:

1 100—𐎶 𐎠𐎵 𐎶 𐎶𐎵
6 500—𐎶𐎶𐎶 𐎠𐎵 𐎶𐎶𐎶 𐎶𐎵
2 000—𐎶𐎶 𐎠𐎵
2 538—𐎶𐎶 𐎠𐎵 𐎶𐎶𐎶 𐎶𐎵 𐎠𐎶𐎶𐎶𐎶
8 000—𐎶𐎶𐎶𐎶 𐎠𐎵

The third expedition statistics

As was seen in sample, numbers such as 65 and 90 are not provided, these numbers were taken from other inscriptions:

6 665—𐎶𐎶𐎶 𐎠𐎵 𐎶𐎶𐎶 𐎶𐎵 𐎠𐎶𐎶𐎶 𐎶𐎶𐎶
31 890—𐎠𐎶𐎶𐎶 𐎠𐎵 𐎶 𐎠𐎵 𐎶𐎶𐎶𐎶 𐎶𐎵 𐎶𐎶𐎶𐎶

In order to better understand Urartian numbering system, it is necessary to give some idea of the Babylonian numbering system. Babylonian arithmetic was created in a plain between the Tigris and Euphrates, which

is called Mesopotamia. It is not only named after geographical location, but also because of a system based on 60.

It is also necessary to express that throughout the ages, different nations settled down in Mesopotamia and have replaced one after the other, then they have created cuneiform writing. Sumerians were the inventors of cuneiform script. Two thousand years before Christ the people of Akkad appeared in Mesopotamia and assimilated Sumer people and adopted their literature and culture and improved them. After Akkadian, Assyrian and Kassites came to the scene and by expansion of the Assyrian power the Babylonian culture was spread in many countries, thus the cuneiform script was being widely used in Southwest Asia and other areas⁶.

Studying 250 Babylonian-Assyrian mathematical cuneiform texts showed that the Babylonians used only two symbols to write numbers and by combining them could write all other numbers. One of the two symbols is like Υ that represents 1 and the other is \langle which indicates 10.

By repeating first digit and decimal in desired number, they could write all numbers up to 59. For example:

$$\Upsilon\Upsilon=2, \quad \langle\langle=20, \quad \langle\langle \quad \Upsilon\Upsilon=23$$

When these symbols were written more than four times, the symbols were written in a more compact way, such as:

$$\Upsilon\Upsilon\Upsilon=5, \quad \langle\langle\langle=50, \quad \langle\langle=40:$$

⁶ - Prof. V. Avriev, History of Ancient East, Yerevan, 1947.

$\gamma = 60.$

$$\Upsilon \Upsilon = 62, \quad \Upsilon \Upsilon \Upsilon = 63, \quad \Upsilon \langle = 70, \quad \Upsilon \langle \langle \langle = 90$$

$\text{YY YY} = 2 \times 60 + 3 = 123$
 $\text{YYY YYY} = 5 \times 60 + 6 = 306$
 $\text{<< YY <<< YY} = 22 \times 60 + 32 = 1352$

Now let's look at the numbers in Urtu system. Urtu also used two marks, one of which was 1 and the other 10. All the numbers from 1 to 99 were demonstrated by combination of these two symbols. 60 was shown by symbol one (such as the Babylonian method) or six nails were presented. From 60 to 99 they were shown by sixties. There was a special mark for 100 (horizontal and vertical nails near each other). To show multiples of a hundred, coefficient was written before the hundred mark. Such as:

$$\text{Y Y } Y = 200, \text{ Y Y Y } Y = 300, \quad \begin{matrix} Y & Y \\ Y & Y \\ Y & Y \end{matrix} Y = 900$$

Similarly they continued up to 1000.

1000 had special symbol, such as:

$$\text{𐎶𐎶𐎶} \text{ 𐎠𐎺} = 3000$$

Similarly, the numbers were shown up to 10,000. This number was just presented as 10 times 1000, which is the product of 10 multiplied by 1000.

$$\text{𐎠} \text{ 𐎠𐎺} = 10 \times 1000 = 10,000$$

They used the word “atibi” for 10,000.

$$25000 = 2 \text{ atibi}$$

$$\text{𐎶𐎶𐎶} \text{ 𐎠𐎺} \text{ 45000-} \text{𐎠} \text{ 4 atibi } \text{𐎶𐎶𐎶} \text{ 𐎠𐎺}$$

Other than atibi there were three more words:

1- Shusini 2- Meshini 3- Kamani

Thus, an interesting system of calculation is presented from 1 to 60. The marks of 1 and 10 were used and from 60 to 100, system of sixties was applied and method for writing large numbers was through use of tens, but the 100 and 1,000 had special symbols.

Therefore, decimal system was fully respected but system of sixties was not ignored totally. As it was noted in the calculation of the Babylonian sixties up to 60, the numbers were written based on the decimal system. If we call Babylonian system as sixties, we naturally need to consider the decimal system for Urartu.

Since 1, 10, 100 and 1000 had their own symbols, there is no negative impact on the system, because there were no zero notation in Urartu.

As in some tablets of the first millennium BC can be seen, before the invention of cuneiform, round signs were used to write the numbers. In order to act quickly and save space circular signs and symbols were replaced by cuneiforms.

The table on next page shows the nail and round numbers which are taken from "Collection of articles on Mathematics," published in 1927, Leningrad. Professor Vigodsky⁷, Noygebaouer and the Canton⁸ in their books⁹ and researches about the existence of the decimal system and Sixties in Babylon and Assyria, have some notes. Decimal system was being used in Urartu, which was essentially different from the decimal system in Egypt and was closer to the current numbering system.

Now this question arises, that if the current decimal number system may be derived from Urartu system? (In which, unlike the Egyptian system, the principle of addition and multiplication is used).

⁷ - M. Vigodsky, "Arithmetic and algebra in the world", Moscow, 1941, p.61

⁸ - "History of mathematics", Vol. 1, 1880, pp. 70-71.

⁹ - Noygebaouer, "Lectures on the Ancient history of mathematical sciences", Vol. 1, Moscow, 1937, P. 110.

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44
45	46
47	48
49	50
51	52
53	54
55	56
57	58
59	60
61	62
63	64
65	66
67	68
69	70
71	72
73	74
75	76
77	78
79	80
81	82
83	84
85	86
87	88
89	90
91	92
93	94
95	96
97	98
99	100

Round and cuneiform symbols table

Addition and subtraction

In an inscription of Sarduri the Second, the summary of the number of captives and spoils allows us to get an idea of how arithmetic operations were performed in Urartu.

The numbers are summarized in the following inscription¹⁰:

12 000 < <Y- Y Y <Y-
2 500 Y Y <Y- Y Y Y-
23 335 << <Y- Y Y Y <Y- Y Y Y- <<< Y Y
58 100 <<< <Y- Y Y Y Y <Y- Y <-

The summary of all the numbers in the table is¹¹

Addition as per inscription				Addition
8135	3500	1100	12735	12735
25000	10500	6500	46600	—4600 42000
6000	4000	2000	12000	12000
2500	—	—	2500	2500
12300	8525	2538	23335	+ 28 23363
32100	18000	8000	58100	58100

The first three columns in this table show in each of the expeditions, the number of prisoners and how much spoils has been collected. In the fourth column total number of captives and spoils is presented. Fifth column shows number of prisoners and the spoils of the expeditions based on our calculations. The second row at fourth and fifth column shows subtraction

¹⁰ - "Encyclopedia of Mathematics", Moscow, 1951, p. 49.

¹¹ - On the tablet it is carved 22000 instead of 12000.

of 46600 and 4200 (if it was 15,000 rather than the 10,500, in this case the difference would be 100) and in the fifth row of columns subtraction of 23335 and 23363 equal to 28 is given. This error should have taken place due to human error or done by engraver. These examples show that large numbers of several thousand have been added easily in Urartu, in this case it is concluded that the arithmetic in Urartu was relatively in an advanced level for its time. By having decimal number system, they could easily perform addition and subtraction, multiplication and division.

Section 4

Armenian Mathematical Sciences after Urartu until the seventh century AD

After the extinction of the Araratyan dynasty known as Urartu (seventh century BC) their territory fell under the reign of the Medes and the Achaemenids and in the fourth century BC. Seleucids were dominant on that land. During the reign of Seleucid, Armenia was divided into three large territories: Great Hayk (Great Armenia), Hayk Minor (Armenia Minor) and Sophene. As per economic growth in 3-4 centuries BC new cities of Armavir, Yervandashad, Yervandakerd, Arshamashad, etc. were built. In the third century BC Hellenic culture was spread all over the East and Armenians were not exempted from this phenomenon.

Armenians under the leadership of Artashes I in 189 BC overcame the Seleucid dominance successfully and announced their independence on the large part of their lands. In 166 BC Artashesian dynasty founded Artashad, one of the oldest cities on the plains of Ararat. Armenian capital was founded on the commercial road to China. This phenomenon caused further development of material and spiritual culture of the era¹².

Artashesian glorious days at the time of Tigran the Great (95-55 BC) reached its peak. His capital Tigranakerd which was called after his name was built, which with Artashad became well-developed cities in the international scene. As a result, these cities were not only commercial and administrative centers, but also became centers of Armenian-Greek intermingled culture. According to Greek historians, especially these two

¹² - History of Armenia, Multi authors, edited by Academician Prof. M.G. Nersisyan, Translated y Edic Baghdasarian (Ed. Germanic), Vol. 1, Tehran, 1981, pp. 51-66.

cities were centers of arts such as theater and a haven and shelter for many Greek scientists.

Artashesian glory diminished in the first year AD. Since then Armenian Arsacid dynasty took over the reign until the early fifth century AD, economic and cultural development continued and made significant progress, especially in the architecture. The climax of architecture was temple Garni built by the order of Tirdates the First (66-100 AD) which demonstrates the glory of engineering and architecture and catches viewer's eye until now. In the following centuries, the temple of Tekor and Yereruk were built as examples of science and technology, architecture and engineering of 5-4 centuries¹³.



Temple of Garni , 1st centurt AD

Architectural heritage of ancient Armenia, especially after the adoption of Christianity among the Armenians (officially in 301 AD) was passed on to next generations and in later years hundreds of big and small churches

¹³ - Sirarpi Der Nersissian, "the Armenians", Translated by M. Rajab Nia, Tehran, 1978, pp. 88-89.

were constructed, which demonstrated the glorious architecture, arts, engineering and scientific talent of this land.



Tekor church - an engraving from the 1840s, carving on the lintel of the above entrance



Yereru(y)k temple

It was natural that in these conditions, science and technology needed to improve, for which mathematics had a focal role. Improvement of techniques and technology in architecture could not be imagined without

advanced mathematics. In fact, the Armenian trade relations with many different peoples of the ancient world helped Armenians to get familiar with achievement of other people in the areas of weights and measures and computational techniques.

So on the one hand the commercial achievement and on the other hand the development of art and architecture techniques were requiring the Armenians to gain higher level of mathematical sciences. Armenian relations with other peoples helped them progress in astronomy and chronology sciences. Studies on the calendar and astronomical calculation methods show that the Armenians had long been familiar with astronomy and they moved from lunar calendar to solar.

Based on the above evidence, and until the invention of the Armenian alphabet, it can be concluded that knowledge of arithmetic and geometry of the Armenians improved widely and was in advanced level.

The invention of the Armenian alphabet had big role in political and cultural life of Armenians. This phenomenon took place in 405 (according to some other sources in 396 AD), by Mesrop Mashtots. Based on new invented alphabet the Armenian literature was established on a firm and ever-lasting basis so that the great heritage of cultural works is available for the current generation and generations to come¹⁴.

Armenian alphabet not only was a means of creating huge heritage of scientific, literary, philosophical and historical works but also helped the Armenians to translate valuable works from other languages into Armenian, through them they got familiar with culture and civilization and sciences of other nations. Many cultural and scientific books were translated from Assyrian and Greek". Many valuable works masterpieces

¹⁴ -"Prominent figures of the Armenian Culture", by Group of Authors, 5-18 centuries. Translated by Edic Baghdasarian (Ed. Germanic), Vol.1, Tehran 1982, pp. 13-28.

were created by Movses Khorenatsi (5th century AD), Yeznic Koghbatsi (5th century AD.), Davit Anhagh and others.

No works were written on geometry, arithmetic, astronomy and chronology and calendars before the 7th century though there are mentions about natural science in historical and philosophical books written until seventh century, based on them high level of science is noticeable.

For example Yeznic Koghbatsi has mentioned about some cosmological problems:

"... And it is said that the sky is spread on the top and also the same underneath and other sides. Earth is surrounded by water and air. Air, water and ground are surrounded by fire. Moon has no light of its own, but its light is from Sun¹⁵".

"They say that the moon and all the stars are lower, when they are directly against the sun, the solar eclipse happens".

Yeznic Koghbatsi, in other parts of his works talks about earth, sky and the moon and their movement and how day and night happens.

Davit Anhagh great philosopher of 5-6 centuries has dedicated chapter 12 of the book "The definitions" to the mathematical sciences. Here, he explains the history of mathematics from ancient times:

"They are also associated with mathematics. The fourth season is about to start and we will state who the creators of these ideas are. We need to know that mathematics was created by Phoenicians, because they were traders who needed intensive computational work. As for music, people of Thrace created it, because there Orpheus was from there and it is said that he is inventor of music.

¹⁵ -Yeznic Koghbatsi, "About rejection of the sects", Yerevan, 1994, p. 162.

But astronomy was established by Chaldeans, because they always had cloudless and clear sky, and so they were able to easily follow and understand the movements of the stars. Egyptian invented geometry due to their needs, because when the Nile flooded and border of the fields got lost Egyptians argue with each other for the boundaries, until they could find a measure named Spanak, which was so that they could find borders.

And here also emerged geometry ¹⁶.

In the ancient world, the assignment of mathematical sciences to a specific nation is not right, because each nation according to the daily needs of computational techniques used it in one way or another. Use of weights and measures in everyday life was the reason for development of computational techniques. Much information on weights and measures among Armenians in the time of Byzantine and Sassanid era dominance has been revealed to us and in this regard, comprehensive research has been conducted by scientists¹⁷.

The numbers and figures which have been used in the works created before the 7th century AD allow us to judge about counting systems since the collapse of Urartu up to the seventh century AD.

Counting system

Armenians have been using alphabet letters to indicate numbers, the Greeks used the same way. Alphabet was used in the sense that the letters were set up in order. This method of counting did not exist from the beginning of human creation until 6-5th centuries BC when Greeks began using this method. But before the first century AD, Attic counting system (or Athenic) was common.

¹⁶ - Davit Anhaght, "The book of definitions", Constantinople, 1791, pp. 160-173.

¹⁷ - Edic Baghdasarian (Ed. Germanic), "Weights and measures in Armenia", Arax Monthly, No. 79, Tehran.

Attic numerals, which were later adopted as the basis for Roman numerals, were the first alphabetic set. They were acrophonic, derived (after the initial one) from the first letters of the names of the numbers represented. They ran $\text{I} = 1$, $\text{V} = 5$, $\text{X} = 10$, $\text{H} = 100$, $\text{K} = 1000$, and $\text{M} = 10000$. 50, 500, 5000, and 50000 were represented by the letter P with minuscule powers of ten written in the top right corner: P^{a} , P^{b} , P^{c} , and P^{d} . The same system was used outside of Attica, but the symbols varied with the local alphabets: in Boeotia, V was 1000.

The present system probably developed around Miletus in Ionia. 19th-century classicists placed its development in the 3rd century BC, the occasion of its first widespread use. More thorough modern archaeology has caused the date to be pushed back at least to the 5th century BC, a little before Athens abandoned its pre-Euclidean alphabet in favor of Miletus's in 402 BC, and it may predate that by a century or two. The present system uses the 24 letters adopted by Euclides as well as three Phoenician and Ionic ones that were not carried over: digamma, koppa, and sampi. The position of those characters within the numbering system imply that the first two were still in use (or at least remembered as letters) while the third was not. The exact dating, particularly for sampi, is problematic since its uncommon value means the first attested representative near Miletus does not appear until the 2nd century BC and its use is unattested in Athens until the 2nd century AD. (In general, Athens resisted the use of the new numerals for the longest of any of the Greek states but had fully adopted them by AD c. 50.)¹⁸

¹⁸ - Heath, Thomas L. *A Manual of Greek Mathematics*, pp. 14 ff. Oxford Univ. Press (Oxford), 1931. Reprinted Dover ([Mineola](#)), 2003. Accessed 1 November 2013.

The Athenian method reminds Roman numeral notation method. With the difference that the latter method the principle of subtraction is used. For example, ":

$$\text{VI} = 6, \text{VIII} = 8, \text{III} = 3, \text{IV} = 5 - 1 = 4, \text{IX} = 10 - 1 = 9$$

As mentioned, the Armenian alphabet system was used to represent numbers, on the other hand, we noted that the Armenian alphabet was created in the late fourth century AD. It should be noted that before that date the Armenian literature should have been created with some other Armenian letters, otherwise we can not imagine how the new Armenian alphabet could generate so rich and comprehensive collection of classical literature in Armenian language (Grabar). So, for sure, "An alphabetical counting system in the era before the advent of the current alphabet existed. But, unfortunately, we do not have sufficient historical resources to support this idea.

So, first 9 letters of the Armenian alphabet represents the numbers 1 to 9 and then the next 9 characters represent tens (10-90), next 9 letters refer to hundreds (100 to 900) in the same way the next 9 letters show thousands (1000-9000) . This numeric system is used even today in Armenian texts. To separate numerical value from letters of the alphabet a line was drawn over them or put point before and after them. (Like .վիւզ. or $\overline{\text{վիւզ}}$. equal to 3043, 3000 = .զն., or 3000 = $\overline{\text{զ.}}$). There were no letters or signs for zero. 0 did not exist since there was no need of zeroes in non-positional

-Thompson, Edward M. *Handbook of Greek and Latin Palaeography*, p. 114. D. Appleton (New York), 1893.

-The Packard Humanities Institute (Cornell & Ohio State Universities). *Searchable Greek Inscriptions*: "[IG I³ 1387](#)" [also known as IG I² 760]. Accessed 1 November 2013.

-Jeffery, Lilian H. *The Local Scripts of Archaic Greece*, pp. 38 ff. Clarendon (Oxford), 1961.

system. Armenians used the word "Byur" (in Pahlavi or Middle Persian it is *bivar* with the same meaning) meaning 10 thousand, in their work. To show large numbers, additional signs were used :

$$\text{ւ} = 10,000 = 10^4$$

$$\text{ժ} = 10 \times 10^4$$

$$\text{հ} = 100 \times 10^4 \dots \text{ք} = 9000 \times 10^4$$

To show the fractions a ten thousand mark () or a crooked line (Slash /) from the left was written, then denominator was written and the numerator of which was always assumed to be 1.

$$\text{՛ք} = 1/8 \text{ (ք} = 8\text{)}$$

$$\text{՛հզ} = 1/23$$

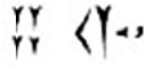
c mark was used for 1/2:

$$\text{զ.ճժ՛հ՛հք} = 3 + 1/2 + 1/10 + 1/40 + 1/88 = 3 \frac{7}{11}$$

As mentioned above, like the Greeks, the Armenian alphabet was used to represent numbers, but there was a difference between them, because the Armenian alphabet consists of 36 letters while Greek alphabet has 27 letters, so Armenians could show numbers up to 9999 without using any symbols, while Greeks could write up to 999 without additional signs by their alphabet.

Although at the time of Urartu Armenians used cuneiform letters, and after invention of current alphabet by Mesrop Mashtots new letters were being used as numbers, but we should note that addition and subtraction of numbers were performed in the same way. They, in any case, used subtrahend before minuend such as the Babylonians.

Armenians at the time of Urartu showed 4000 as:



But after the invention of new alphabet 4000 was shown as: ԴՆ.=4000

Another example:

3900 = .ԳՌԹՃ.

Comparing these figures shows that the proposed method is still the same.

Section 5

Anania Shirakatsi, a great scientist of the 7th century AD and the importance of his book “Arithmetic”

About the life and time of Anania Shirakatsi¹⁹

In the seventh century AD, the region was experiencing great historical developments, but despite the invasions of the Byzantine and Arabs, economics and science were booming in Armenia. Armenia participated actively in the international trade and urban reconstruction, especially construction of new churches, during his period, had significant growth and progress. In the light of these developments, favorable conditions existed for the prosperity and progress of science and technology.

No criticism can be made on the architecture of the period, including temples and churches of Mren, Talin, Talish and Zvartnots, for these and other monuments like them, are brilliant examples of engineering design and construction techniques in the Armenian architecture with genuine oriental Armenian characteristics.

Zvartnots temple could only be constructed, if there were accurate mathematical and geometrical calculations and plans. This temple has unique architecture style because there is no second architectural work like this in the Armenian territory. Researchers across the East and West have done extensive studies, but have not found any similarity between the structure of this temple and other architectural works. The temple is built

¹⁹ - Complete works and autobiography of Shirakatsi have been translated from Armenian into Persian in four volumes by me and the first two volumes have been published by ARC, the rest will be published gradually. I hope to have enough support to publish the same volums also in English, Ed. Baghdasarian.

probably between 644 to 652 AD and the architect was Nerses Tayetsi and its executive director was Hovhan²⁰.



Zvartnots temple

The distinctive feature of the architecture of this temple with works of other lands is not the construction power applied in building it, (like the pyramids and other monuments in which pieces of 20 cubic meter rocks have been installed at a height of 30-40 meters) and not the elegant engravings on the stone, but what is much weird and wonderful is that Armenian architecture has four stone columns with a diameter of 0.82 meter, and two columns with a diameter of 0.60 meter are connected to each other on the arc, so heavy weight of several thousand tons is hanging and this could not be possible other than through a precise mathematical calculation.

It is worth mentioning that Armenian territory played an important transit role in international trade, including the export/import of Chinese silk and valuable goods from China, Central Asia, North India and the West.

²⁰ -Toros Toramanian, "Some subjects about the History of the Armenian Architecture", Yerevan, 1942, p.249.

In these conditions, the Armenians could not be indifferent to their business in the global market with their products and industries such as salt, iron, copper, fruit, fish, cotton, paint, carpet and other imported commodities.

The economic boom caused harmoniously development and progress in the sciences. Construction of buildings for which compass, ruler, mechanical devices were used, excessively required the ability to perform accurate calculations, using weights and exact values, true knowledge of environmental and natural sciences, especially arithmetic and gravity rules led to a higher degree of progress.

In the seventh century, one of the most prominent figures of science the Armenian mathematician Anania Shirakatsi was born. He, who was called "Calculator" (in Armenian Hamarogh) or the mathematician, is one of most famous and prominent thinkers and figures in natural science. We have reliable information about the life and activities of Shirakatsi, as we have his "Autobiography", according to which, He was born in the first quarter of the seventh century in the village of Ani, located in the historic Province of Shirak. He gained his primary education in church school of the village, then for higher education he travelled to Western Armenia, which at that time was under Byzantine rule. He especially loved mathematics which he believed that was the mother of all sciences. He studied at Constantinople and Trebizond with the famous scientists of the time (Tyukhik). At that time the school had a great reputation and a large library including many books on history, medicine, yearbooks, works remained from the Paganist times, works by the known and unknown.

Shirakatsi's date of death is not known. In the book "History" which belongs to him and is available to us, he mentions an invasion of one of the kings of Khazars to Armenia in the year 685. Due to the fact that at this time he should have spent much of his life, it can be inferred that he

should have died during 80's of the seventh century. Researchers consider 685 as his death date²¹.

UNESCO included Anania Shirakatsi's 1400th anniversary in the list of its important anniversaries of 2012 (Mesrop Mashtots 1650th and Sayat-Nova's 300th anniversaries also are in this list). The proposal was submitted on behalf of the Byurakan Astrophysical Observatory. Anania Shirakatsi (612-685) was the greatest Armenian scientist of the Middle Ages. He was a philosopher, mathematician, cosmologist, chronologist, the founder of exact and natural sciences in Armenia. Having advanced astronomical views for his epoch and having left rich astronomical heritage, Shirakatsi is, by right, considered to be the founder of astronomy in Armenia. His scientific heritage has significantly influenced on further development of the Armenian natural science²².

After returning home country, he developed scientific and educational activities. At the end of 630s he founded a natural science mathematical high school, where he paid special attention to the teaching of four sciences (arithmetic, music, geometry, and astronomy), as well as teaching of geography, chronology, philosophy, and the Armenian language. Shirakatsi compiled the work "Knnikon", separate parts of which: "Cosmology", "Chronology", "Map", the textbook of arithmetic, etc. were later observed as independent works.

Shirakatsi is a supporter of the cosmological demonstration of God's existence. In his opinion God has not created the whole variety of the world but four elements (fire, air, earth, water). Each of them has double qualities, and the interaction of these elements is the basis of the origin

²¹ - Prof. Ashot Abrahamian, "Bibliography of Anania Shirakatsi", Yerevan, 1944, p. 206.

²² -Areg Mickaelian, IAU WG "Astronomy and World Heritage" member, N E W S L E T T E R OF THE ARMENIAN ASTRONOMICAL SOCIETY (A r A S) No. 55 (May 26, 2012)

and the development of the material world. Shirakatsi investigates the elements and the qualities physically and cosmologically, with the cosmological regard to the observation he comes to the problem of the structure of the Universe.

In his opinion the Universe is limited, it has two limits; small (the Earth) and large (the upper sky). The upper invisible sky is spherical; it is in the state of endless motion of excessively high velocity. The continuous motion and the sphericity of luminaries, the Sun, the Earth, the Moon and other celestial bodies situated under the lower visible, as well as under the firmament, are due to the motion and shape of the upper sky. The Earth is situated in the center of the Universe, and its stable balanced state is a result of combination of weight and motion power. There are 7 mobile areas between the firmament and the Earth; the celestial bodies are settled on them and deprived of their own light and heat, get them from the upper sky.

Shirakatsi observed a causal connection between the motion of celestial bodies, the terrestrial space and the processes on the Earth. He grounded the endless and continuous formation and development as a main law of existence and as a reason of nature eternity. Shirakatsi thinks that celestial and terrestrial bodies are formed of the same four elements, which condition the material unity of the Universe. He gave much importance to the experimental cognition, studied the experiment as a means of cognition, a kind of knowledge and a criterion of truth. He proved the truth of his theoretical constructions by observational and experimental examples, natural and simple models (e.g. he described the Universe by the model of an egg). He used scientific methods such as observation, analogy, experiment, etc. He put the problem of consequent change of knowledge, considered the antique science to be the base of Christian world outlook and supported the concept of their non-discrepancy, belief and unity of science.

With amazing physical intuition and critical judgments, he supported the spherical hypothesis of the Earth, suggested correct explanation of the layer of the Milky Way and developed good understanding of the surface of the Moon as a celestial body. He could correctly explain the eclipses of the Moon and the Sun and had a number of other advanced astronomical views for that period of time.

An anonymous chronology is related to Anania Shirakatsi, which is compiled on the basis of Armenian and a number of foreign resources. His metrological, meteorological works, and works devoted to precious stones have scientific interest. He criticized astrology, fatalism and witchcraft. His scientific heritage had a significant influence on further development of the Armenian natural science thought, a number of Byzantine scientists' views as well. Shirakatsi considerably enriched the Armenian and at the same time, the Eastern musicology.

Shirakatsi compiled chronological tables, astronomical textbooks, etc. He left a number of books and letters, which are preserved till nowadays. Shirakatsi's works are considered the main resource for the explanation of Armenian ancient astronomical terminology, including the names of constellations and stars. Tyukhik received the young Armenian with pleasure and began to teach him. Shirakatsi remembers his teacher with respect and warmth: "He loved me as his own son, he writes, and he passed all his knowledge with such eagerness that my class-mates were jealous".

Shirakatsi had been at Tyukhik's school for 8 years; he became proficient in exact science and came back to his native land with rich knowledge base. Here he opened a school and devoted himself to teaching and research. He wrote research works in Astronomy, Mathematics, Geography and in other fields of science. Among medieval sources there is evidence that in 667 – 669 Shirakatsi first, on the instructions of

Catholicos Anastas, formed a new Armenian Calendar with the aim to make a fixed one.

Among Shirakatsi's works, the most valuable and unique one is the textbook in Arithmetic. It contains addition, subtraction, multiplication tables, "6-hazariak", 24 problems and 8 entertainments. Shirakatsi also left a rich heritage on Calendars. His works on Calendarology are of great scientific-historical value.

Among Shirakatsi's works an important one is considered "Knnikon", which contains "Easter Speech", "Christmas Speech" and "Chronicle" including synchronized spatial tables of changeable and fixed calendars for 532 years. Among Shirakatsi's calendaric works another important one is "Patchen Tomari", which is of great value not only on the history of Armenian Calendars but also on the calendars of neighbor nations: such as Egyptians, Jews, Syrians, Persians, Romans, Georgians, Macedonians and others. In this work a calendar is included, which has a close relation with other calendar systems.

Shirakatsi formed a large number of tables and calendar cycles. Among the tables more attractive ones are "Tables of Lunar Cycle" where the exact date of new moon and full moon of the nineteenth-year cycle is given, i.e. the year, the day and the time (in hours and minutes) of new moon and full moon occurring is discovered. And as the nineteenth-year phases of the moon are always repeated then all these tables can be used all the time.

Shirakatsi's works on Cosmography are of great value. The basic questions on Exact Sciences are included in it. These works give us an opportunity to learn about his views on Exact Sciences.

Following antique scientists, Shirakatsi thinks that the perceptible world and all its substances consist of the four really existing elements: land,

water, air and fire. In his opinion the world is “a certain composition of mixed elements”. Shirakatsi imagines the nature in the process of movement and change. The existing old education decomposes with the time and instead a new one occurs. On the basis of many examples from real life considered as a proof of it he comes to the following philosophic scientific conclusion. “Existence is the beginning of extinction and the extinction, in its turn, is the start of existence and as a result of this non-harmful contradiction the world continues to exist.

Shiraktsi’s point of view on Cosmography is also significant. The question related to the earth shape interested the humankind for a long time. Various approaches were expressed in different time periods. In his cosmographic works Shirakatsi gives a peculiar explanation: “I think the earth is of an egg-shaped form, the ball-shaped yolk is in the middle, white is around it and the shell surrounds everything; in the same way, the earth is in the center like the yolk, air is around it like the white and the sky surrounds everything like the shell.

Shirakatsi’s astronomical system is not heliocentric but geocentric. Accepting the egg-shape of the Earth it was important to explain the issue of earth balance. This question was of great interest since the ancient times and different opinions were made; some people thought that the earth lay on a gigantic elephant others considered that it lay on a huge whale, on seas, etc. Shirakatsi gives a very original explanation to this question. He finds that it is balanced by two opposite forces and he writes: “The Earth tends to go down with all its weight and the wind tries to raise it up with all of its power. That’s why the earth doesn’t fall down and the wind doesn’t raise it up”. In his cosmographic work Shirakatsi tells about the Galaxy (the Milky Way) and tries to explain its main point. Criticizing all the legends of his time he gives a scientific explanation concerning that issue. According to him, the Galaxy is the same as “the mass of densely possessed and weakly shining stars”. Shirakatsi absolutely rejects

conservative scientists' points of view. Among them there are church priests who think that the moon has its own light. He finds that the moon doesn't have its own light and obtains light from the Sun which reflects the light of the ether like a mirror. According to him it is related to the reflection of sunlight and the change of lunar phases. The sun is in the fifth zone of the sky, and the moon is in the 4th one. Therefore, the moon obtains the light from above, and as the sun and the moon are in perpetual motion round the earth at different speed they either approach or move away. During the period when the sun approaches the moon, its light cycle begins to diminish and in case of moving away it begins to enlarge.

Shirakatsi explains high and low tide of the oceans and seas by changes of lunar phases: during the full moon water level in the oceans and seas begins to rise and when diminishing to fall. In his cosmographic work Shirakatsi discusses also the solar and the lunar eclipses. According to him the solar eclipse occurs when the sun is in the northern hemisphere, and the Moon in the southern, the earth is between them and hinders the light penetration to the Moon then the lunar eclipse occurs.

One of his valuable works is "Ashkharhatsuyts" ("Geography"), which includes the description of all the countries in the world discovered at that time. That is a great work in the World Geographical and Cartographical literature of that time. "Ashkharhatsuyts" consists of Introduction and two parts. In the introduction Shirakatsi determines the zones and temperature of the Earth defining the points of view existing in the science. In the first part he gives a general description of the Earth and in the second part there is a description of different countries known at that time. In the main part of "Ashkharhatsuyts" is the description of all the countries in the three continents known then: they are Europe, Africa and Asia. Here the author defines the boundaries of each described country, distinguishes the rivers, mountains and towns, etc.

Metrological works of Anania Shirakatsi are also of great interest: “about the length measures” and “about the weight measures”. His metrological works contain the basic measurements and concepts existing in Armenia then. It is notable that in the 7th century Armenia was an important center of international trade.

Status of arithmetic prior to the seventh century AD

In order to get familiar with “arithmetic” by Shirakatsi, we need to understand the status of math books before the seventh century AD. First, we need to talk about the Greek arithmetic. Greeks do not deny the fact that in this area generally they were dependent on Egyptians and Babylonians they learned many things from them and improved their science to a higher level.

Greeks by the development of mathematics got it to a very high and ideal degree and reached a high scientific level. Scientists like Euclid, Archimedes, Apollonius (third and second centuries BC) were prominent figures for Alexandrian mathematical books. The "Principles" of Euclidean geometry consists of 13 books. Arithmetic meant science of numbers among the Greeks and they studied the properties of numbers, classifying them as odd and even, Real and Complex. Greeks generally considered arithmetic as a theoretical science. An applied arithmetic was a different subject and was known as Logistic. Plato accepted this classification and this classification was done probably before him. The first arithmetic book was compiled about 100 AD by Nicomachus²³ a Greek philosopher and mathematician who lived in the first century AD.

²³ - The *Treviso Arithmetic* is the earliest known printed mathematics book in the West, and one of the first printed European textbooks dealing with a science. The *Treviso Arithmetic*, or *Arte dell'Abbaco*, is an anonymous textbook in commercial arithmetic written in vernacular Venetian and published in Treviso, Italy in 1478.

In order to review the arithmetic book of Shirakatsi we need a quick look at the Nicomachus arithmetic. It consists of two books, the first book odd or even and real or complex numbers are presented. Then different categories of even numbers are discussed. In the second book polygonal numbers have been discussed in details. Based on this mathematician, numbers are linear, flat, triangular, quadrilateral and so on. Comparisons have three categories: arithmetic, geometry and harmonics. It was used for centuries as a source and was translated into Arabic in the ninth century.

After Nicomachus, in the third and fourth centuries AD, Diophantus famous mathematician of the second school of Alexandria, compiled a book titled "Arithmetic" which has mostly algebraic content.

In the history of mathematics in particular, we need to mention "Mathematical collection" by Pappus (fourth century) which consists of 8 books, including all of the noteworthy achievements of arithmetic and geometry.

The works of Theon of Smyrna and Iamblichus on the arithmetic are important source of mathematics.

As far as we know in the seventh century AD (the period in which Shirakatsi lived), there were no mathematical school in the East.

The fall and division of the Roman Empire was catastrophic for the second school of Alexandria, as the scientific life was stopped there and it was transferred to Athens and lasted one hundred years (until the end of the sixth century).

After the collapse of the school of Athens, Byzantine school was founded in the seventh century which survived until the 15th century AD when Empire was conquered by the Turks.

None of these schools introduced famous mathematicians to the world. Athenian scholars studied and interpreted the works of authors of Greek authors and scholars of the Byzantine school spent their time on theology and grammar issues, they almost did not pay attention to the natural sciences.

Thus, in the seventh century AD, there were no particular mathematical schools, so that certain mathematician could be attributed to them.

Contents of "Arithmetic" by Anania Shirakatsi

Development of natural sciences in Armenia (7th century AD) had significant impact on the arithmetic works, cosmology, and the calendars, weights and measures. Textbook of "Arithmetic" by Shirakatsi has a long educational and instructive history. First the arithmetic problems are very important and include 24 questions. The problems have been published in various languages several times by the researcher in different places.

Unfortunately, the full text of book of arithmetic by Shirakatsi has not been left for us. The researchers believe that 24 questions and answers are only a portion of the total problems. This matter was found in light of finding other parts of the book. In introducing the great thinker of the seventh century, armenologists such as Galust Megerdichian, G. Khalatian, academician Hovsep Orbeli, P. Ardruni, Professor Ashot Abrahamian, G. Petrosian and others have worked.

The sections of the book that have been kept up to our times include: 1-Addition and subtraction tables 2- Multiplication tables, 3- Six-thousands 4- Problems and answers 5- Mathematial entertainment.

Addition tables

These tables begin with the following introduction:

"Beginning of Anania Shirakatsi's speech, *Chapter 1*

You lovers of wisdom and science who are with me, as a caring and nice teacher I'm going to explain to you the techniques of arithmetic which emerged as the voice of ancestors. Learn my tables. Although I have summarized extensive texts to be not boring to you, but to some extent I have also made them easy and simple in order that you can learn them completely. Now I start from the most simple and low level and I have taken to account the scientific level of children and those who are not familiar to the subject. Primary instruction of arithmetic which is called addition (*endouneloutyoun*²⁴)".

After the introduction, the tables are listed, as example we present the following four groups.

As mentioned before Armenians have used the Armenian alphabet to show numeric values since old times. As we see Shirakatsi also used this method.

²⁴ - Armenian term.

ա ա բ	1+1=2	ն ն ս	1000+1000=2000
ա բ գ	1+2=3	ն ս լ	1000+2000=3000
ա գ դ	1+3=4	ն լ ա	1000+3000=4000
ա դ ե	1+4=5	ն ա բ	1000+4000=5000
ա ե զ	1+5=6	ն բ գ	1000+5000=6000
ա զ ի	1+6=7	ն գ լ	1000+6000=7000
ա ի ղ	1+7=8	ն լ փ	1000+7000=8000
ա ղ թ	1+8=9	ն փ բ	1000+8000=9000
ա թ ժ	1+9=10	ն բ ա'	1000+9000=10000
բ բ դ	2+2=4	ս ս ա	2000+2000=4000
բ գ ե	2+3=5	ս լ բ	2000+3000=5000
բ գ զ	2+4=6	ս ա գ	2000+4000=6000
բ ե ի	2+5=7	ս բ լ	2000+5000=7000
բ զ ղ	2+6=8	ս գ փ	2000+6000=8000
բ ի թ	2+7=9	ս լ բ	2000+7000=9000
բ ղ ժ	2+8=10	ս փ ա'	2000+8000=10000
բ թ ժա	2+9=11	ս բ ա'ն	2000+9000=11000

In order to understand these tables we need to refer to the following list of letters and their equivalent numbers²⁵ (Capital and small letters respectively) :

²⁵ - Manuscript No. 1770, Page 375, Manuscript Library of Mesrop Mashtots (Matenadaran), Yerevan.

Ա ա=1	Ժ ժ=10	Ճ ճ=100	Ռ ռ=1000
Բ բ=2	Ի ի=20	Ս ս=200	Ս ս=2000
Գ գ=3	Լ լ=30	Յ չ=300	Վ վ=3000
Դ դ=4	Խ խ=40	Ն ն=400	Տ տ=4000
Ե ե=5	Ծ ծ=50	Շ շ=500	Ր ր=5000
Զ զ=6	Վ վ=60	Ո ո=600	Յ չ=6000
Է է=7	Հ հ=70	Չ չ=700	Ի լ=7000
Ը ը=8	Ձ ձ=80	Պ պ=800	Փ փ=8000
Թ թ=9	Ղ ղ=90	Ջ յ=900	Ք ք=9000

To show larger numbers of letters the sign ^ was put on top of the letters and increased the value by ten thousand times greater than the amount of letters such as Ճ=100, if we put the sign, it will refer to one million. Fractions were demonstrated with special symbols and half value was shown with a symbol like C.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

A sample from the tables of arithmetic of Shirakatsi text No. 1770,
Manuscripts Library Matnadaran- Yerevan.

Subtraction

Subtraction tables begin with the following introduction:

"Aania Shirakats's promise to his students, *Chapter II*

For you lovers of deep thinking and science and technology education, I explain the subject as following. A great workout is the second subject which is called subtraction (*Bats-droutyoun*²⁶)".

Immediately after the introduction, the subtraction tables are presented, Here we show four of them as examples, two tables with old symbols and two tables with new signs²⁷:

ա ծ ք	10—1=9	ժ ց ղ	100—10=90
ա ք ը	9—1=8	ժ ղ ձ	90—10=80
ա ը Է	8—1=7	ժ ձ ց	80—10=70
ա Է զ	7—1=6	ժ ց Կ	70—10=60
ա զ Ե	6—1=5	ժ Կ ծ	60—10=50
ա Ե ղ	5—1=4	ժ ծ Խ	50—10=40
ա ղ զ	4—1=3	ժ Խ Լ	40—10=30
ա զ ք	3—1=2	ժ Լ Ի	30—10=20
ա ք ա	2—1=1	ժ Ի ժ	20—10=10

հ ռ շ	1000—100=900	ն ա' ք	10000—1000=9000
հ շ ալ	900—100=800	ն ք փ	9000—1000=8000
հ ալ շ	800—100=700	ն փ լ	8000—1000=7000
հ շ ռ	700—100=600	ն լ ց	7000—1000=6000
հ ռ շ	600—100=500	ն ց ր	6000—1000=5000
հ շ հ	500—100=400	ն ր ա	5000—1000=4000
հ հ յ	400—100=300	ն ա վ	4000—1000=3000
հ յ լ	300—100=200	ն վ ա	3000—1000=2000
հ լ հ	200—100=100	ն ա ն	2000—1000=1000

²⁶ - Armenian term.

²⁷ - Manuscript No. 1770, Page 387, Manuscript Library of Mesrop Mashtots (Matenadaran), Yerevan.

Addition tables consist of four groups for the digits, tens and hundreds. Total number of combinations in the tables is 180. Number of subtraction tables is 36 and each contains 9 combination and total is 324. Addition and subtraction tables show that at the time of Shirakatsi no signs were being used. The signs (+ and -) were applied from the 15th century on. The equal sign (=) is being used from the sixteenth century.

A page of subtraction tables by Anania Shirakatsi, No. 1770 version Matenadaran

We notice that subtrahend was written first. It is interesting that a few centuries BC, the Babylonians did the same but with cuneiform. Number of the numbers used in the addition and subtraction counted up to 11,000.

Multiplication

Multiplication tables begins with the following introduction:

"Anana Shirakatsi's advice to his students. *Chapter III*.

Those who have deep thoughts in numbers need to boldly follow my instructions such as light blue waves and short in front of the high-speed ships that are indifferent to the events around them. The benefits are obtained by hard work and tireless efforts.

The third exercise. Third chapter which is called multiplication (*Bazmapatik*²⁸) ”.

After this introduction, multiplication tables are offered. We mention some of them with old and new symbols:

²⁸ - Armenian term.

A sample page of Multiplication tables, Anania Shirakatsi, Text No. 1770 Matenadaran

ա	ա	ա	$1 \times 1 = 1$	ք	ա	ք	$9000 \times 1 = 9000$
ա	բ	բ	$1 \times 2 = 2$	ք	բ	ա' փ	$9000 \times 2 = 10^4 + 8000$
ա	գ	գ	$1 \times 3 = 3$	ք	գ	բ' լ	$9000 \times 3 = 2 \cdot 10^4 + 7000$
ա	դ	դ	$1 \times 4 = 4$	ք	դ	գ' ց	$9000 \times 4 = 3 \cdot 10^4 + 6000$
ա	ե	ե	$1 \times 5 = 5$	ք	ե	դ' ր	$9000 \times 5 = 4 \cdot 10^4 + 5000$
ա	զ	զ	$1 \times 6 = 6$	ք	զ	ե' տ	$9000 \times 6 = 5 \cdot 10^4 + 4000$
ա	է	է	$1 \times 7 = 7$	ք	է	զ' լ	$9000 \times 7 = 6 \cdot 10^4 + 3000$
ա	ը	ը	$1 \times 8 = 8$	ք	ը	է' ս	$9000 \times 8 = 7 \cdot 10^4 + 2000$
ա	ր	ր	$1 \times 9 = 9$	ք	ր	ը' ն	$9000 \times 9 = 8 \cdot 10^4 + 1000$
ա	ն	ն	$1 \times 1000 = 1000$	ք	ն	շ'	$9000 \times 1000 = 900 \cdot 10^4$
ա	ս	ս	$1 \times 2000 = 2000$	ք	ս	ա' պ'	$9000 \times 2000 = 1000 \cdot 10^4 + 800 \cdot 10$
ա	վ	վ	$1 \times 3000 = 3000$	ք	վ	ա' բ'	$9000 \times 3000 = 2000 \cdot 10^4 + 700 \cdot 10$
ա	տ	տ	$1 \times 4000 = 4000$	ք	տ	վ' ռ'	$9000 \times 4000 = 3000 \cdot 10^4 + 600 \cdot 10$
ա	ր	ր	$1 \times 5000 = 5000$	ք	ր	ա' շ'	$9000 \times 5000 = 4000 \cdot 10^4 + 500 \cdot 10$
ա	ց	ց	$1 \times 6000 = 6000$	ք	ց	բ' ց'	$9000 \times 6000 = 5000 \cdot 10^4 + 400 \cdot 10$
ա	լ	լ	$1 \times 7000 = 7000$	ք	լ	գ' յ'	$9000 \times 7000 = 6000 \cdot 10^4 + 300 \cdot 10$
ա	փ	փ	$1 \times 8000 = 8000$	ք	փ	ւ' զ'	$9000 \times 8000 = 7000 \cdot 10^4 + 200 \cdot 10$
ա	ք	ք	$1 \times 9000 = 9000$	ք	ք	փ' ց'	$9000 \times 9000 = 8000 \cdot 10^4 + 100 \cdot 10$
ա	ա' ա'		$1 \times 10000 = 10000$	ք	ա' ք'		$9000 \times 10000 = 9000 \cdot 10^4$

Multiplication tables consist of 36 groups and each composed of four tables, in three tables 9 products and in one table 10 products are listed. Therefore, in every group there are 37 products ($37 = 10 + 9 \times 3$), and 1332 products are included in total ($1332 = 37 \times 36$).

The largest product of the multiplication tables is 10×9000 . Usage of these kinds of numbers especially in the lower grades, shows the level of the arithmetic in the seventh century AD in Armenia.

Nowadays, any student who knows the multiplication table is able to do multi-digit multiplication, but the situation in Armenia was different in old times. Armenians in the seventh century, when multiplying numbers did

not keep one digit in mind and write the other, but they multiplied the numbers at once, and performed addition later and they had to know not only the multiplication table (for 1 to 9) but also many other products such as $100 = 10 \times 10$, $1000 = 100 \times 10$, etc. That is why Shirakatsi had prepared the tables for his students. To multiply 1,560 by 1,560 they performed the following steps:

1560

1560 \times

1000000 500000 60000

500000 250000 30000

60000 33600

Total 2433600

It seems from the above tables that they multiplied 1000 separately by 1000, 500, 60 and 500 in 1000, 500, 60 and 60 by 1000, 500 and 60, and then they added them up.

Six-thousands (Vets-hazaryak²⁹) or the inverse or opposite values table

Six-thousands of Shirakatsi with old and new display are as follows:

ա	ց	ց	1	6000	6000
բ	վ	ց	2	3000	6000
գ	ս	ց	3	2000	6000
դ	ռ	ց	4	1500	6000
ե	ւ	ց	5	1200	6000
զ	ն	ց	6	1000	6000
է	պ	ց	7	875	6000
ը	լ	ց	8	750	6000
թ	հ	ց	9	667	6000
ժ	ն	ց	10	600	6000

This table continued up to the last letter in the Armenian alphabet at that time, (9000 = Ջ).

As shown in Table, the numbers are arranged in such a way that in each case the number obtained is 6000. It starts with 1 and continues until 9000. Shirakatsi's opposite table is arranged for 6000, these systems of sixties existed in Babylon.

²⁹ - Armenian term.

Two pages of tables of division "Dark, "Kotchatk" and "Six-thousands" of Anania Shirakatsi

Tables of inverse values were used as the auxiliary sources for performing divisions.

As per manuscript No. 1973 in Matenadaran we can conclude that Shirakatsi or his students prepared division tables for five thousand, four

thousand, three thousand etc. the same such as six-thousands. For example, two tables are presented here:

ա	բ	ք	լ	5000	5000	ա	ա	ա	1	4000	4000
բ		ս	2		2500	բ		ս	2		2000
գ		առկգ	3		1666	գ		առկգ	3		1333
դ		աւծ	4		1250	դ		ա	4		1000
ե		ա	5		1000	ե		չիւ	5		740
զ		ալիւ	6		840	զ		առի	6		670
է		չծէ	7		717	է		չռի	7		570
ը		աիէ	8		627	ը		չ	8		500
թ		չծ	9		580	թ		ռիւ	9		440
ժ		չ	10		500	ժ		ռ	10		400
ի		ւծ	20		250	ժա		—	11		—
						ժբ		յիւ	12		305
						ի		ւ	20		200

As can be seen on the table provided, Tables of inverse values over time evolved into the tables of divisions. In Six-thousands table of Shirakatsi if we exchange position of product and coefficient we can obtain division table.

Before the six-thousands tables there are two tables titled "Dark" and "Kotchatk". "Dark" means "even" and "Kotchatk" means "odd". This table is displayed in old and new styles as follows:

բ	դ	զ	ը
2	4	6	8
ի	իւ	կ	ճ
20=2·10	40=4·10	60=6·10	80=8·10
ւ	ն	ա	ալ
200=2·10 ²	400=4·10 ²	600=6·10 ²	800=8·10 ²
.....

մին		նին			նին	
$200 \cdot 100 \cdot 1000 \cdot 1000 = 2 \cdot 10^{10}$		$400 \cdot 100 \cdot 1000 \cdot 1000 = 4 \cdot 10^{10}$			$600 \cdot 100 \cdot 1000 \cdot 1000 = 6 \cdot 10^{10}$	
նին		նին			նին	
$600 \cdot 100 \cdot 1000 \cdot 1000 = 6 \cdot 10^{10}$		$800 \cdot 100 \cdot 1000 \cdot 1000 = 8 \cdot 10^{10}$				
ա	գ	ե	զ	ը	թ	
1	3	5	7	9		
ժ	Լ	Ժ	ճ	Դ	Ե	
$10 = 1 \cdot 10$	$30 = 3 \cdot 10$	$5 \cdot 10$	$7 \cdot 10$	$9 \cdot 10$		
Բ	Դ	Զ	Ը	Ժ	Լ	
$100 = 1 \cdot 10^2$	$300 = 3 \cdot 10^2$	$5 \cdot 10^2$	$7 \cdot 10^2$	$9 \cdot 10^2$		
.....						
նին		նին			նին	
$1 \cdot 10^{10} = 100 \cdot 100 \cdot 1000 \cdot 1000$		$3 \cdot 10^{10} = 300 \cdot 100 \cdot 1000 \cdot 1000$				
նին		նին			նին	
$5 \cdot 10^{10} = 500 \cdot 100 \cdot 1000 \cdot 1000$		$7 \cdot 10^{10} = 700 \cdot 100 \cdot 1000 \cdot 1000$				
նին		նին			նին	
$9 \cdot 10^{10} = 900 \cdot 100 \cdot 1000 \cdot 1000$						

As can be seen in the table by even numbers 2, 4, 6 and 8 and the odd numbers 1, 3, 5, 7 and 9 lines are formed in horizontal rows as arithmetic progression and vertical line as geometric progression.

Questions

A chapter of the arithmetic of Shirakatsi is entitled "Questions and Answers" (or "problems and solutions") consisting 24 arithmetic problems. The answers to these problems are presented without explaining how to solve. Shirakatsi's problems with the wording of today are displayed as the following equation:

- $\frac{1}{2}x + \frac{1}{4}x + \frac{1}{11}x + 280 = x,$
- $\frac{1}{2}x + \frac{1}{4}x + 24 = x,$
- $\frac{1}{2}x + \frac{1}{4}x + 421 = x,$
- $\frac{1}{5}x + \frac{1}{10}x + 240 + 2000 = x,$

5. $\frac{1}{4}x + \frac{1}{8}x + 150 = x,$
6. $\frac{1}{5}x + \frac{1}{15}x + 110 = x,$
7. $\frac{1}{2}x + \frac{1}{4}x + \frac{1}{7}x + 45 = x,$
8. $(x + 15)50 = 80x,$
9. $\frac{1}{4}x + \frac{1}{10}x + \frac{1}{20}x + \frac{1}{90}x + 210 = x,$
10. $\frac{1}{4}x + \frac{1}{6}x + 140 = x,$
11. $x \left[1 - \left(\frac{1}{2} + \frac{1}{3} \right) \right] \left[1 - \left(\frac{1}{2} + \frac{1}{3} \right) \right] \left[1 - \left(\frac{1}{2} + \frac{1}{3} \right) \right] = 11,$
12. $\frac{1}{3}x + \frac{1}{4}x + \frac{1}{6}x + \frac{1}{7}x + \frac{1}{28}x + 3 = x,$
13. $x \left[1 - \left(\frac{1}{2} + \frac{1}{4} \right) \right] \left[1 - \left(\frac{1}{2} + \frac{1}{4} \right) \right] \left[1 - \left(\frac{1}{2} + \frac{1}{4} \right) \right] = 5,$
14. $\frac{1}{3}x + \frac{1}{6}x + \frac{1}{14}x + 54 = x,$
15. $\frac{1}{4}x + \frac{1}{7}x + \frac{1}{11}x + 318 = x,$
16. $(x + 39)140 = 218x,$
17. $x \left(1 - \frac{1}{2} \right) \left(1 - \frac{1}{5} \right) \left(1 - \frac{1}{8} \right) \left(1 - \frac{1}{7} \right) = 7200,$
18. $\frac{1}{3}x + \frac{1}{4}x + \frac{1}{5}x + \frac{1}{6}x + 210 = x,$
19. $2[2(2x - 25) - 25] - 25 = 0,$
20. $\frac{1}{2}x + \frac{1}{4}x + \frac{1}{12}x + 360 = x,$
21. $x \left(1 - \frac{1}{2} \right) \left(1 - \frac{1}{7} \right) \left(1 - \frac{1}{8} \right) \left(1 - \frac{1}{14} \right) \left(1 - \frac{1}{13} \right) \left(1 - \frac{1}{9} \right),$
 $\left(1 - \frac{1}{16} \right) \left(1 - \frac{1}{20} \right) = 570,$
22. $a_n = \left[100 : \frac{(1+10)}{2} 10 \right] n, n = 1, 2, 3, \dots, 10,$
23. $80x = 200 \cdot 414720 = 82944000,$
24. $x + \frac{x}{2} + \frac{x}{3} = 1,$

Shirakatsi's mathematical problems can be categorized in the following groups:

Group 1: Problems No. 1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 14, 15, 18 and 20 are as follows:

$$(1/a)x + (1/b)x + (1/c)x + \dots (1/k)x + m + n = x$$

Group 2: Problems No. 8 and 16 as $(x + a)b = cx$.

Group 3: Problems No. 11, 13, 17 and 21 are as follows:

$$x [(1 - (1/a + 1/b)) [1 - (1/a + 1/b)] \dots [1 - (1/a + 1/b)]] = A$$

Or

$$x (1 - 1/a) (1 - 1/b) (1 - 1/c) \dots (1 - 1/n) = B$$

Group 4: Problem No. 19 just $2 [2 (2x - a) - a] - a = 0$

Group 5: Problem No. 22 as $ak = [100 / (1 + 10) / 2] kk = 1, 2, 3, \dots$

Group 6: Problem No. 23 as $ax = bc$

Group 7: Problem No. 24 as $x + x/a + x/c = 1$

Based on the similarities of majority of equations and the fact that the numerators are only 1, we can conclude that these 24 problems are only part of the problems of Shirakatsi.

Shirakatsi has written his book with great patriotism. For example, problems No. 1 and 8 are presented here:

First problem: I heard from my father that when Armenians were fighting against Persians, great gallantries were done by Zorak

Kamsarakan. He attacked the army of Persians three times in a month. At the first time he slaughtered a half of the army. At the second time he slaughtered a quarter of the army. At the third time he slaughtered $1/11$ part of the army. All the rest (280 men) fled to Nakhijevan. Now, with the number of the rest you must count the number of the Persian army.

Eighth problem: When Armenians were in revolt against Persians, Zorak Kamsarakan killed Suren. One of noble men of Armenians was sent as ambassador to the Persian king to tell him about the killing. He [The ambassador] was passing 50 miles in a day. After 15 days Zorak Kamsarakan got to know about that so he sent his men to catch the ambassador. They were passing 80 miles in a day. Now, you must simplify, when they caught the ambassador.

Shirakatsi's solution to the mathematical problems can be applied even nowadays in the schools. Shirakatsi has been able to show the fractions by using special symbols.

Methods of solving arithmetic problems by Anania Shirakatsi

It seems that Shirakatsi used "hypothetical method" to solve the problem, for example to solve problem No.1 we assume that the number of enemy soldiers was 44. We chose 44 because that's the smallest number divisible by 2, 4 and 11. If we divide the number by 2, 4 and 11 number of the killed can be obtained in the first, second and third times:

The first time $22 = 44/2$

The second time $11 = 44/4$

Three times $4 = 44/11$

Sum of $22+11+4= 37$

Therefore, the number of enemies killed is 37 and number of survived soldiers is equal to $44-37=7$, which is clearly wrong because it is clear that the problem stated that the 280 people have survived, the exact number of the soldiers we divide 280 by 7 and multiply the result by 44.

$$280/7=44$$

$$44 \times 40 = 1760$$

So the total number of enemy soldiers is 1760.

By using the same method problems 2, 3, 4, 5, 6, 7, 9, 10, 12, 14, 15, 18 and 20 will also be solved.

Some of the problems can be solved by other methods such as reverse (inversion) or the "red numbers" which were also used by the Egyptians.

Interestingly, many of the problems in arithmetic can be found in the problems of Anania Shirakatsi.

Mathematical entertainment

As part of the Shirakatsi's arithmetic there is a chapter titled "**Mathematical entertainment**". He explains that he wrote them as you eat and drink you can read them and get entertained.

From Shirakatsi's recreational mathematics only No. 1, 2, 3, 4, 5, 6, 8 and 9 have remained. The lack of the entertainment 7 shows that we have not achieved complete book of Shirakatsi's arithmetic. We have presented complete text of them in Annex No. 2 of our present book. .

These types of problems exist among other nations as well. These jokes make fun besides the progress and development of the mathematical knowledge. By the second problem, adults can measure children's mathematical talent.

The mathematical tables which are available to us and also great heritage of the books, collections, the great works of architecture and engineering in general are evidence for existence of high level of mathematics and science, computational techniques in ancient Armenia.

Anania Shirakatsi's textbook in arithmetic in difference with Nicomachus includes rich contents in the field of computational techniques, it is the oldest textbook we have received through the history and in the absence of similar works, it is of great importance for the history of mathematics, particularly arithmetic.

Section 6

Leon, the famous mathematician of the ninth century AD.

About Leon (Levon) the mathematician comprehensive information is not left, even in resources of the ninth century there are a few hints about a few people with the name of Leon. But without doubt one of them is the mathematician Leon that we are trying to to give some details about his life and works.

Based on researches done up to now and according to the testimony of existing resources, mathematician Leon was born in the first decade of the ninth century AD, also lived and worked in this century. He then acquired his education in Constantinople and Andros Island then got busy on scientific research and teaching. Around the year 840 Leon was appointed as Bishop of Thessaloniki and in the year 843, when the images of the saints once again became common in prayers, Leon and his uncle Hovhannes as opponents were removed from church. Therefore, Leon turned back to scientific work and teaching. At the beginning he started teaching at the school of "Church of forty children" then he was appointed director of new established University of Constantinople. Most probably he died in the year 870.

The years 866-856 were the best and brightest scientific time for career of Leon and the university under his management. Leon had famous students from whom we can name Theodor in teaching geometry, Theodegios in the field of astronomy, Komitas in grammar. Leon was also teacher of Cyril the inventor of the Cyrillic.

One of the important works that have significant knowledge about Leon in the field of science education, mathematics is "The blame book»

(Parsavagirk³⁰) written by his student Constandin which is the best criterion to assess Leon's works.

After accurate and complete investigation on "The blame book", we realized what a great service to the development of the physical sciences and mathematics Leon has demonstrated at the University of Constantinople.

Teaching geometry, arithmetic, astronomy and music had special and important place in the curriculum of the university. Constandin's virulent blame was because Leon had given priority to the natural sciences in the unpleasant time of the Middle Ages.

Constandin considers his teacher Leon as a friend of Euclid and Ptolemy. He also in his other citations states that Leon was a mathematician.

Other than "The blame book" another book entitled "Philosopher Leon's believes in Christ and denial of the gods of pagan" has reached to us, probably written by the student Constandin including critical materials about Leon.

One piece of work is remained under "philosopher Leon about solar eclipse and lunar eclipse" which is assumed to belong to mathematician Leon, but by studying its contents, it can be concluded that it can not be possibly related to mathematician Leon and the "Leon" referred should be another personality with the same name³¹.

Magistros Simeon a Byzantine historian in his book has citations about Leon's knowledge about mathematics-physics, as follows:

³⁰ - Armenian term.

³¹ - N. Adonts, "History researches", Paris, 1948, p. 527-528.

"Leon philosopher who was living in Thessaloniki recommends the Kaiser Theophilos to provide two similar clocks and put one of them in a castle near Tarson in the land of Cilicia and another one to be installed in the palace. There were written some events on one of them that occurred in the Syria (or supposed to happen). Twelve incidents have been recorded by warning signs (fire) and notified to neighboring fortresses until it reached the palace..."³².

Since fire alarms and signs were not visible during daytime, so they needed to be transmitted with 12 hours delay during night time and this delay could have unrecoverable consequences. In order these optical signs to be more comprehensive Leon used them in short and long intervals and a combination of them. By this technique, Leon founded a conventional system for transmitting the signs. As per the notes of Michaelis Clycus Leon had invented a device to reproduce the sound of birds and lions to draw people's attention. The device, which was installed in the palace Magnavra, was being demonstrated for the ambassadors and representatives of other countries.

As per the above-mentioned and historians' notes, the efforts of Leon made considerable progress in the fields of mathematics, mechanics and acoustics.

³² - "History of Natural sciences and techniques in Armenia) Yerevan, 1960, Vol. 1, p. 15-16.

Section 7

Grigor Magistros, translator of the oldest Armenian version of Euclid's geometry

Grigor Magistros and his time

After two centuries of Arab rule, in the fourth quarter of the ninth century Armenians by heroic struggles of Bagratuni Dynasty were able to overthrow the shadow of the Arabic dominance and create independent Armenian kingdom (886-1045 AD). During a century of peaceful conditions from the second quarter of the tenth century to the fourth decade of the eleventh century under the rule of Bagratunis, economic and cultural life and the reconstruction had significant development and progress. Trade between East and West in 11-10 centuries prepared the base for the growth and development of cities. Especially the perpetual conflict between Byzantine and Arabs in the South caused the international transit road to transfer to the North toward Armenia.

In the 11-10 centuries in parallel with the growth and development of the ancient cities such as (Dvin, Archesh, Khlat, Van, new towns such as Ani, Kars, Ardsn, especially in the way of transit road between Dvin and Trabzon were established. Ani which was mentioned in the sources as a fortress became the capital of the kingdom in 953 by Ashot III and after a short time turned to one of the largest, most prosperous, richest, most commercial and beautiful architecture of the Near East. The cities of Kars, Ardsn and Khlat received the same fate.

The economic boom created proper conditions for reconstruction and development activities in architecture, engineering and various fields of science. Building of churches, palaces and mansions, castles, bridges,

schools, etc. was encouraged. These changes required development and promotion of science, so schools and universities were established³³.

In parallel with the development of science, history and literature of the natural sciences, including mathematics recorded achievements and was promoted to the higher level and in such circumstances, one of the prominent figures of Armenia, Grigor Magistros entered the scene of sciences.

Grigor Magistros was born in the last quarter of the tenth century, around the year 990 AD. His father Holum Vasak was younger brother of Vahram Pahlavuni commander-in-chief of the Armenian army.

Grigor during his study period and the years after, studied the Armenian and Greek sources and learned almost all sciences available at that time and obtained multilateral knowledge. As per his letters, he was proficient at Mathematical Sciences, Medicine, Armenian and Greek literature, philosophy, rhetoric, theology and so on and as a prince he had also passed military training.

Grigor's father got killed by invasion of Delmic people in 1021 AD. So he became the head of the dynasty. His commanded construction of many buildings and palaces during 1013-1051, so that many of them still remain today.

Magistros as one of the most prominent commanders military men played a major role in the political life of his time. After the overthrow of the kingdom of Bagratuni, he gave up his inherited lands in Bejni to the Byzantine Empire and in return received cities and villages in Mesopotamia. What is of importance regarding Magistros is his activities in the field of historiography, translations, science and education. Like the

³³ - "History of Armenia", Translated by Edic Baghdasarian (Ed.. Germanic), Tehran, 1981, pp. 193-208.

Greek philosophers, he had his own university, where he taught science had lectures.

Grigor Maistros has some works and translations of which his "Letters" is important, in each of them he speaks about natural sciences, particularly mathematics. One of his works is the interpretation of grammar, poetry, a verse in 10-16 lines. He has left translations in various fields of science. The Armenian translation of the geometry of Euclid is very important which is made from the Greek text. As a scientist, philosopher and orator Magistros is one of the few figures in the 11th century who was not clergyman. He died in 1058 was buried in the present day Hasanqaleh castle (A city in Western Armenia, in the province of Erzurum, Bassen region, 40 km East of Erzurum)³⁴.

³⁴ - "Prominent Figures...", Yerevan, 1976, pp. 2013-223. Also Armenian Encyclopedia, Vol 6.

The earliest translations of Euclid

Ancient Greek culture does not have any field of science as rich as mathematics. Greek mathematical heritage standardized this field, the principles and definitions and terminology, mathematical and scientific methods of research over many centuries. Modern world is enjoying this scientific heritage.

Euclid is one of the prominent figures in mathematical field in school of Alexandria (third century BC). He has left us a valuable and famous work "Elements" which is foundation of geometry until now. The book has been translated into many languages of the world. "Elements" after the Bible, perhaps more than any other book in the history of the West, has been reprinted and studied. Since the invention of printing in Europe, it has had over a thousand versions and even before Printing Industry comes to life, its manuscripts were widely distributed and been used as textbook for teaching geometry.

Until the ninth century Europeans only knew the name of the Boethius but Euclid was little known to Europeans. In the late eighth and early ninth century the translation of Euclid was done by Assyrian physicians. The book was first translated into Assyrian, then into Arabic. Father and son Ben Isaac were the translators who due to their lack of familiarity with mathematics their translation was revised by Ben Qara between 892-902³⁵.

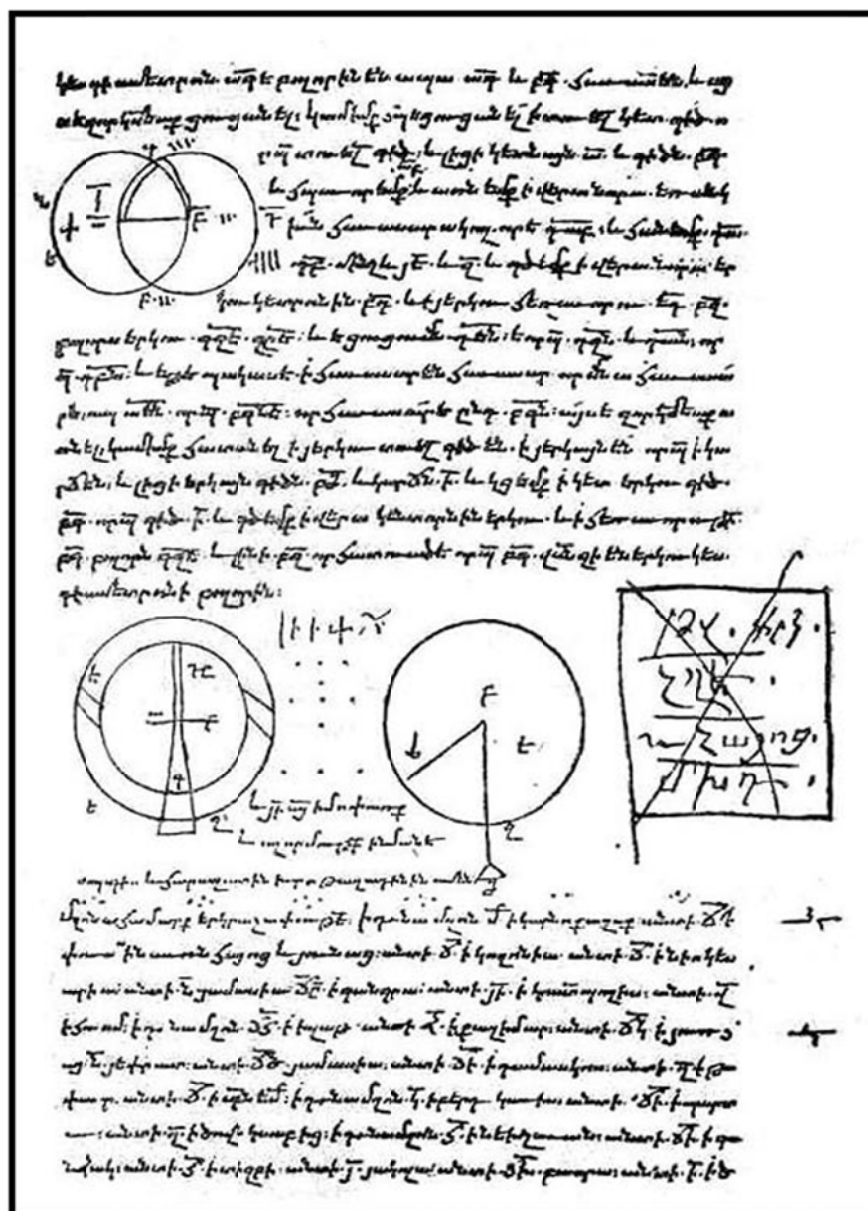
Euclid, for the first time in Western Europe, was translated in 1120 by Athelarde from an Arabic version into Latin. It was the first time translated partially from the Greek to Latin by Lambertus, in 1501 and was published in 1505 in Venice. Full text of Euclid geometry was translated from Greek in 1533.

³⁵ - G. Petrosian, "Armenian Mathematics in Middle ages", Yerevan 1959, p. 100.

Armenian translator of Euclid and His Time

For the first time, Grigor Magistros, himself as the Armenian translator has mentioned about Armenian translation of Euclid's Geometry. In this regard, Sahak Prunian an Armenian mathematician of 18th century pointed it out in "List of books translated into Armenian language" (1825), and mentioned Magistros's name as translator of Euclid geometry. Armenian modern Haykazian dictionary (1837) has references in this regard. Professor Ashot Abrahamian in his doctoral thesis (1943) proved that Grigor Magistros is the translator of Euclid's geometry and attribution to other authors was no longer correct.

Regarding the time of the Armenian translation of this work we should consider the date of a letter that Magistros wrote to Sarkis Vartapt, in which he has mentioned about the translation of Euclid. The researchers believe the letter was written in 1051, so 1051 should be the date of translation. A part of the Armenian translation of Euclid (copied in the 13th century) is stored in Matenadaran of Yerevan under the number 4166. There are also parts of it at the University of Tübingen and Venice. In the 4166 version, definitions, axioms and theorems of the first book have been preserved.



One of the pages of the Armenian translation of geometry - 4166 version Matenadaran

A brief Comparison of Greek and Armenian texts

Euclidean geometry

Armenian translation of Euclid is done directly from the Greek text. The comparison of the two texts shows that they generally correspond with each other, but the differences between them are as follows:

1. Definition 5 and 6 of the Greek text ("Two ends of line have points" and "end of the surface is a line") is missing in the Armenian text.
2. The third definition of the Armenian text reads: "and located between two points", which is a straight line. Such a definition does not exist in the Greek text.
3. The definition (2) in the Greek text reads: "Straight line is a line between (all) the point equally located. This definition is equivalent to the definition No 4 in the Armenian text.
4. The fourth definition of Armenian ("shortest line between two points is a straight line") is missing in the Greek text.
5. The Greek definition 7 "flat surface is the one located equally between straight lines", in the Armenian text as: "flat surface that is on the front line, which are against each other".
6. In Greek, "If the lines that make an angle are straight, it is called right angle" (definition 9) in the Armenian is as: "and if the vertical line is straight, it is called perpendicular" (definition 9).
7. Greek: "Limit is the boundary between the farthest end of everything." (Definition 13). Armenian: "and the border or limit is the peripheral of everything." (Definition 13).

8. Greek: "A shape is formed by one or more limits." (Definition 14). Armenian: "Shape is the one among one or more borders." (Definition 14).

9. Greek: "a semi-circular shape is the one limited by diameter and its center is the center of the circle." (Definition 18).

Armenian: "semi-circular shape is the one limited by diameter and arc." (Definition 18)

10. Greek: "vertical shapes are those limited by vertical line. Triangles, squares, polygons are limited by three, four and more straight lines." (Definition 19).

Armenian: "vertical shapes are those surrounded by vertical straight lines. And triangles by three sides, squares by four straight line and more of them are limited by more straight lines". (Definition 19).

11. Greek: "of the shapes the one is square that has equal sides and equal angles." (Definition 22). Armenian: "and there are shapes that have four sides, equal sides and right angles in the middle is the square....". (Definition 22).

12. Greek: in the axioms word "necessary condition" in Armenian: "What they think is possible is five."

13. In the Armenian text, "and if equally reduced from inequality, remaining is also unequality." (Article 5). This principle does not exist in the Greek text.

Grigor Magistros continues and completes methods of Shirakatsi

In the history of medieval Armenian Magistros is known as the most prominent figure in Mathematical Sciences after Shirakatsi. This is for two reasons, First he is the translator of Euclid's geometry and second, he was the first person who discovered the value and importance of Shirakatsi's works. It was Grigor Magistros who revealed the works of Shirakatsi out of obscurity and set them in math textbooks for schools and universities at the time.

Grigor Magistros studied the works of Shirakatsi since childhood until his old age, for that reason he mentions about Shirakatsi with great desire and respect, and in one of his letters to Catholicos Petros expresses his excessive anger for negligence to that great scientist and his works.

Grigor Magistros not only in math, but also in other fields continued Shirakatsi's way, this is obvious in his letters. In his idea teaching should be done in this way: First study the Bible and the myths and proverbs then study parts of Homer and Plato then study "Four techniques of arts" which means arithmetic, music, geometry and astronomy, only after this process one can pass to the next stage of education, including grammar and other subjects. Magistros believed that without the "Four Arts" it was impossible to understand the philosophy.

This method is mentioned in his letter No. 45 to his students Barsegh and Yeghis. He taught in Armenian language and non-translated, local works were also used as textbooks.

Section 8

Hovhannes Sarkavak and "Polygonal numbers" Life and Works

The second half of the 11th century coincided with the devastating inroads and invasions of Seljuk Turks. They threaten the peace of the entire region, towns and villages, people's homes were leveled, including the cities of Armenia Ardsn and Ardske located in the North of Lake Van. A large part of the cities Manazgerd, Kars, Archesh and some other cities of Armenia were plundered and ruined. It was natural that in such circumstances, the peaceful economic, political and cultural life to be disrupted. Since the late 11th century the shadow of terror and destruction of Seljuks across the Armenian land gradually disappeared and in the early 12th century Seljuk Empire was divided into some local states such as emirates of Erzurum, Kars, Khlat, Erzincan, Dvin. Kurdish Shadadians bought Ani from Alp Arslan in 1071 and expanded their dominance to Shirac. Following the strengthening of Georgia, the North East Armenia became autonomous by Armenian Zakarian dynasty (Zakarid Armenia 1201-1360) as Commanders of the joined army. Principality of Sassoon and Aghtamar, Syunik, Lori Bagratuni, Moks, Rshtuni maintained their autonomy. Regions under Seljuks lost their economical important, cities and villages were destroyed and fields changed into pastures. The only areas under control of the Armenian princes could maintain their cultural identity and to some extent their economic and political prosperity.

One of the greatest cultural centers of the 11th century was Haghbat Cathedral School in the territory of the Bagratuni kings of Lori that Hovhannes Sarkavak was its prominent personality.



Zakarid Armenia (1201–1360)

Hovhannes Sarkavak was born around the years 1050-1045 in a religious family in the village of Parisos in the province of Artsakh. He obtained his primary education in native village at Haghbat church school and spent his time not only on religious sciences but also natural sciences and achieved excellent results. Historians and writers such as Giragos Gandzaketsi (13th century) have reported about talent and knowledge of Sarkavak:

“... So diligent that once went to a cave with others with books, at time of exiting he hid in a cave and they closed entrance of the cave. A few days later, they again went to the cave and found him with surprise and asked him how he would survive without water and food, and he responded by pointing out that these are the books that I have used a few days”.³⁶

Hovhannes Sarkavak, after finishing education established a school in the city of Ani then he settled in Haghbat Cathedral. As a result of his hard

³⁶ - Giragos Gandzaketsi, History of the Armenians, Yerevan, 1961, pp. 116-117.

work and efforts he became aware of all sciences of the time and got titles such as "great priest", "wise" and "philosopher". Ganzaketsi in another part of his book writes:

"... Hovhannes Sarkavak was scientifically above all and every thing, he had brilliant and powerful ideas. Because this man was very knowledgeable and had the blessing of God, all his words were wise ... ³⁷".

Hovhannes Sarkavak was famous in the field of science and education and he trained several students in the schools (Haghbat and Ani) whose scientific works remain up to our days. Based on the material that students have about their teacher's notes it could be concluded that he, among other subjects, had taught mathematics. Hovhannes as a progressive scientists of his time fought against ignorance and superstition of the Middle Ages and gave great importance to the teaching of natural sciences. He is one of the first teachers who emphasized the importance of testing and experience in various fields. As we know in Europe for the first time in the 13th century AD R. Bacon pointed out the importance of mathematics in social life and special attention was paid to scientific experiments. Hovhannes Sarkavak was living in the late 11th century and early 12th century about 150 years before Bacon has stated these ideas:

"Without experimentation no acceptable theory can be possible, because it is only the experience that is consistent and indisputable".

He died in 1129 and was buried in the courtyard of the Church of Haghbat which remains until today and on his tomb reads:

"This building belongs to the Sarkavak the Philosopher".

Hovhannes Sarkavak works are in many fields such as church, poetry, history, chronology, astronomy and arithmetic, which unfortunately only a

³⁷ - Girakos Gandzaketsi, History of the Armenians, Yerevan, 1961, p. 113.

part of them remains today. But this much of his works that came to us is sufficient to judge about him and his works and knowledge. He is one of the most educated persons of his time, capable and master teacher, poet, mathematician and philosopher.

Among his numerous works on mathematics, the "Polygonal numbers" is important.

About the author of "polygonal numbers" and of Armenian manuscripts

A few pieces of "Polygonal numbers" are available in different versions of the library of manuscripts in Matenadaran. Here we present three more complete versions. One of them is No. 4150 version of the 13th century, which contains tables and the title is: "The numbers resulting from the geometric polygonal shapes by Hovhannes the priest". The numbers presented in the following table is noteworthy that the signs of the Armenian alphabet (with their values) are used as numbers:

Հոսանքի թվերի հարկածային խմբակները									
Խմբակ	1	2	3	4	5	6	7	8	9
1	1	3	6	10	15	21	28	36	45
2	3	6	10	15	21	28	36	45	55
3	6	10	15	21	28	36	45	55	66
4	10	15	21	28	36	45	55	66	78
5	15	21	28	36	45	55	66	78	91
6	21	28	36	45	55	66	78	91	105
7	28	36	45	55	66	78	91	105	120
8	36	45	55	66	78	91	105	120	136
9	45	55	66	78	91	105	120	136	153
10	55	66	78	91	105	120	136	153	171
11	66	78	91	105	120	136	153	171	190
12	78	91	105	120	136	153	171	190	210
13	91	105	120	136	153	171	190	210	231
14	105	120	136	153	171	190	210	231	253
15	120	136	153	171	190	210	231	253	276
16	136	153	171	190	210	231	253	276	300
17	153	171	190	210	231	253	276	300	325
18	171	190	210	231	253	276	300	325	351
19	190	210	231	253	276	300	325	351	378
20	210	231	253	276	300	325	351	378	406

The first "polygonal numbers" Hovhannes Sarkavak, manuscript No. 4150, Matenadaran

Numerical interpretation of the above table are as follows:

Triangular:

Armenian Alphabetical number: ա բ գ դ ե զ լ ղ թ ժ ժա ... իա

Triangular: ա գ գ ժ ժե իա իը լզ խե ծե կզ ... վա

Triangular:

Sequential numbers: 1 2 3 4 5 6 7 8 9 10 11 ... 21

Triangular: 1 3 6 10 15 21 28 36 45 55 66 ... 231

Quadrilateral:

Armenian Alphabetical number: ա գ ե է թ ժա ժզ ժե ժէ ժթ իա ... իաա

Quadrilateral: ա դ թ ժզ իա լզ իթ կդ ծա ճ ճիա... նիաա

Quadrilateral:

Sequential numbers: 1 3 5 7 9 11 13 15 17 19 21 ... 41

Quadrilateral: 1 4 9 16 25 36 49 64 81 100 121 ... 441

Numbers in the table continue to 15 polygonal. It should be mentioned that the tables do not contain any geometrical shapes.

In the version of the year 1589 under No. 1170 two-pieces remain. The first piece has no title and is on only one sheet that on one page appears the following table, we present the modern numbers immediately after the table:

The first page of the first piece of version 1170, Matenadaran

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

Աղյուսակ № 3

Աղյուսակի համար	Երեսի համար	Առաջին տողի համար	Երկրորդ տողի համար	Յոթերորդ տողի համար	Յոթերորդ տողի համար	Յոթերորդ տողի համար	Յոթերորդ տողի համար	Յոթերորդ տողի համար	Յոթերորդ տողի համար	Յոթերորդ տողի համար	Յոթերորդ տողի համար	Յոթերորդ տողի համար	Յոթերորդ տողի համար	Յոթերորդ տողի համար	Յոթերորդ տողի համար	Յոթերորդ տողի համար	Յոթերորդ տողի համար	Յոթերորդ տողի համար
1	1	1	1	1	1	1	1	1	1	2	2	1	8	1	1	1	1	1
2	3	3	4	4	5	5	6	6	7	4	6	3	18	9	3	2	4	4
3	6	5	9	7	12	9	15	11	18	6	12	5	24	18	9	4	16	16
4	10	7	16	10	22	13	28	16	34	8	20	7	32	27	27	8	64	64
5	15	9	25	13	35	17	45	21	55	10	30	9	40	36	81	16	256	256
6	21	11	36	16	51	21	66	26	81	12	42	11	48	45	243	32	1024	1024
7	28	13	49	19	70	25	91	31	112	14	56	13	56	54	729	64	4096	4096
8	36	15	64	22	92	29	120	36	148	16	72	15	64	63	2187	128	16384	16384
9	45	17	81	25	117	33	153	41	189	18	90	17	72	72	6561	256	65536	65536
10	55	19	100	28	143	37	190	46	235	20	110	19	80	81	19683	512	262144	262144
11	66	21	121	31	176	41	231	51	286	22	132	21	88	90	59049	1024	1048576	1048576
12	78	23	144	34	210	45	276	56	342	24	156	23	96	99				
13	91	25	169	37	247	49	325	61	403	26	182	25	104	108				
14	105	27	196	40	287	53	378	66	469	28	210	27	112	117				
15	120	29	225	43	336	57	435	71	540	30	240	29	120	126				
16	136	31	256	46	376	61	496	76	616	32	272	31	128	135				
17	153	33	289	49	423	65	561	81	697	34	306	33	136	144				
18	171	35	324	52	477	69	630	86	783	36	342	35	144	153				
19	190	37	361	55	533	73	703	91	874	38	380	37	152	163				

1 = Vertical 2 = Additional triangular square 3 = Right Squar
 4 = square right 5 = Extra pentagon 6 = pentagon
 7= hexagon 8 = Hexagonal 9 = Extra Heptagon
 10=Heptagon 11= Darc 12 = Darc phrase
 13 = Kuchatk 14 =Kvayk 15 = Separate Kvayk
 16= 3 multiplier 17 = two multiplier 18= four multiplier

In the second piece which has the name of Hovhannes Sarkavak on it, has two tables including numbers to polygonal fourteen . We present one sample of each as follows:

Triangular		Quadrilatera			Number Generator triangular	Number Generator quadrilateral
1	1	1	1		1	1
2	3	3	4		2	3
3	6	5	9		3	5
4	10	7	16		4	7
5	15	9	25		5	9
6	21	11	36		6	11
7	28	13	49		7	13
8	36	15	64		8	15
9	45	17	81		9	17
10	55	19	100		10	19
11	66	21	121		11	21
12	78	23	144		12	23
13	91	25	169		14	25
14	105	27	196		14	27

15	120	29	225		15	29
16	136	31	256		16	31
17	153	33	289		17	33
18	171	35	324		18	35
19	190	37	361		19	37
20	210	39	400		20	39
21	231	41	441		21	41
22	253	43	484		22	43
23	276	45	529		23	45
24	300	47	576		24	47
25	325	49	625		25	49
26	351	51	676		26	51

In the version of the year 1445 under Number 8973, the tables of polygon numbers are shown with calendar data:

Եռանկյան թվեր	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
Չորսանկյան թվեր	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
Հինգանկյան թվեր	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
Ճիշտ թվեր	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
Երեքանկյան թվեր	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
Տասնանկյան թվեր	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
Ճիշտ թվեր	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12
	1	2	3	4	5	6	7	8	9	10	11	12

A page of polygon numbers (version 1770).

Content and potential resources of "polygon numbers" of Hovhannes Sarkavak

The first systematic study of mathematics which deals with number theory belongs to Nicomachus, Greek mathematician of the first century AD (his book called "Introduction to Arithmetic")³⁸. This book was used as textbook for theory of numbers more than a thousand years. Boethius in the fifth century AD wrote a book based on the book of Nicomachus as one of the classic books on mathematics which was used extensively.

Hovhannes presents his works only in tables and does not provide an explanation along with them. The possible sources of the tables are probably the mathematical works of Anania Shirakatsi especially because Grigor Magistros, in one of his letters, mentions that large volumes of Anania Shirakatsi's works were available and being used in the eleventh century.

Again, it is possible that he had access to the works of Philon, Nicomachus or Boethius and later authors and used them for educational purposes and scientific research. These are partly determined by comparing their works.

Having studied polygon numbers of Hovhannes Sarkavak and his possible sources, general extraction methods of these numbers are determined.

Triangular numbers derived from sum of the natural sequence of numbers. Since the natural numbers form an arithmetic progression, so to obtain the three terms of Kth we should have the following:

$$a_k = (k + 1) / 2 \cdot k$$

³⁸ - D. Estervick, Concise history of Mathematics, Tehran, 1987, p. 74.

By putting 1, 2, 3, 4, 5, 6 instead of K numbers 1, 3, 6, 10, 15, 21 are obtained.

Similarly, the formula for quadrilateral, pentagonal, hexagonal numbers, and so are acquired.

In general, we should consider the sum of arithmetic progression that the first element is:

1 and common difference is $n=2$:

$$a_k = [2 + (n-2)(k-1)] / 2 \cdot k$$

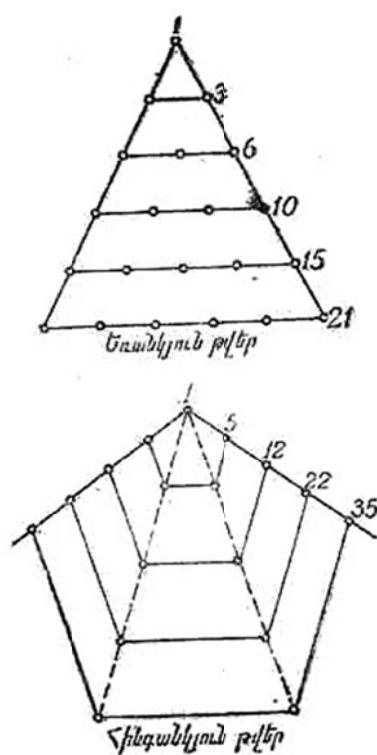
In which:

$$K = 1, 2, 3, \dots$$

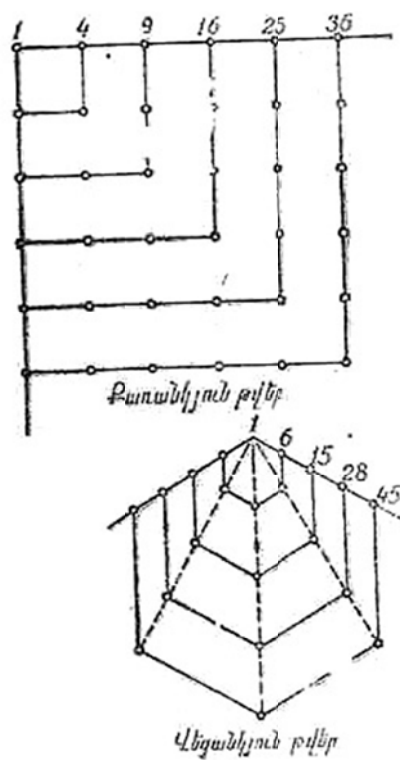
$$n = 3, 4, 5$$

If $n=3$ triangular numbers formula is obtained. If $n = 4$ the formula for quadrilateral numbers is obtained, and so on.

The term "polygon numbers" shows that the tables should have lines and edges of these numbers, but because of neglect of copyists, these lines are not reserved. Diophantus (around 250 AD) helps us to visualize the geometric view of polygonal numbers:



Shape of pentagonal numbers



Shape of hexagonal numbers

Tables of Hovhannes Sarkavak indicate that the Armenians in 11-12 centuries were teaching arithmetic in the schools with similar sense as nowadays and text books were developed based on the basic content of the classical mathematical works. Through works of Sarkavak we are able to be familiar with the Armenian mathematical terminology.

Section 9

Mathematical works of Nikoghayos (Nokolay) Artavazd

Biography of Nikoghayos Artavazd

As previously discussed, in the Middle Ages prominent figures in mathematics such as Anania Shirakatsi, Grigor Magistros, Hovhannes Sarkavak were having creative life in Armenia and compiled their scientific works in the Armenian language, and also translated important foreign sources into the mother tongue and on the other hand they taught in the Armenian schools and universities as math teachers.

However, medieval Armenian mathematical thinking was not limited only to the works of mathematicians. Historical studies show that in the Middle Ages there had been a group of Armenian mathematicians living outside and have created works in foreign languages. They were directly associated with their home country, where they had tried to promote science. On the other hand, along with external sources, they used the works of fellow mathematicians. Nikoghayos Artavazd is one of the prominent figures of this kind. He lived in Byzantine and wrote his scientific works in Greek.

Studying the works of Nikoghayos Artavazd is important for us, as foreign researchers have not mentioned about his Armenian origin, on the other hand his works remind us of Anania Shirakatsi.

Here, we present the topics of Nikoghayos Artavazd's works:

"Explanation and summary of arithmetic prepared in Byzantium by Nikoghayos Artavazd from Izmir, expert in arithmetic and geometry, at

the request of Georg Khachik president of Hall of petitions, were written to be easy to learn, now let's see what is its contents:

O! my dear friend, my faithful Theodore, now Nikoghayos Artavazd writes this from Byzantium to Zabukheh of Klazdman".

Paul Tanner, a French researcher on the basis of documents and manuscripts in the National Library in Paris (Greek manuscripts No. 2428, 2535, 652, 2107, 612) who has published works of Artavazd said: "Artavazd has ... Armenian origin (?) Whose name has mistakenly mentioned as Artavazdis ".

Examining the historical evidences suggests that Artavazd was an Armenian mathematician. Professor Hrachia Atcharian, famous Armenian linguist writes:

1. "Artavazd's name indicates that he must be Armenian for sure. Although this name is originally Iranian, but after the Arab invasion in the seventh century the other Muslim Iranians stopped using such names and this tradition has been maintained only by the Armenians. There is no such a name among the Greeks and it was only the Armenians who had used this name³⁹.

2. Khachatur is a combination of Khach (Cross) + Tur (given) which means granted to/by the cross, the same way as Astvadsatur (divine, granted by God)... and since the 11th century has been used and the its diminutive forms are Khachik, Khachuk and Khacho⁴⁰.

³⁹ - H. Atcharian, A dictionary of the Armenian names, Vol. 1, Yerevan, 1942, p. 316.

⁴⁰ - H. Atcharian, A dictionary of the Armenian names, Vol. 2, Yerevan, 1944, p. 467.

3. Nikoghayos derives from Greek Nicholas that means "victory of the people" and is common among Slavic nations and since the 13th century has also been used by the Armenians⁴¹.

4. Georg, derived from the Greek name Georgios means "farmer". From the 5th century AD on the Armenians are also using it⁴².

Nikoghayos Adonts a renowned Armenian researcher by studying the names mentioned believes that all of them are Armenian.

We have little information about Artavazd. As per his works, he must have been living and working in the 14th century. In one of his calculations it is indicated that in 1341 he was alive. Nikoghayos other than several mathematical works also edited some works in grammar and donated them to his son Pavel Artavazd. A geometric work is also attributed to him which remains in the Paris National Library under Greek manuscripts of Cod. Suppe trec 682.

The content and importance of mathematics articles of Nikoghayos Artavazd

Two articles of Artavazd which are about computing technology can be assumed as two parts of one single work. The first part is an introduction in which the author tries to interpret issues with a method of expression and discusses on the basis of counting and numbers he talks about the quantitative sides and how to show the values and the analysis of problems.

In the first two chapters of the first article he presents the calculation methods and specifies the duties of his audience, he presents decimal

⁴¹ - H. Atcharian, A dictionary of the Armenian names, Vol. 4, Yerevan, 1948, p. 75.

⁴² - H. Atcharian, A dictionary of the Armenian names, Vol. 1, Yerevan, 1942, p. 454-455.

system to calculate alphabetically, showing large numbers using specific symbols. To do this, he uses points and signs. For example:

$$a=1$$

$$a=1000$$

..

$$a=10000$$

..

..

$$a=100000000=10^8$$

In the third chapter, numbers from 1 to 9999 can be explained by the fingers. Part I includes Artavazd's method to show numbers by the fingers. A few examples are presented below:

"Now let us see how to show the numbers with your fingers. The left hand always shows the first digit and decimal numbers, but the right hand is used to express hundreds and thousands. The use of symbols is necessary because the hands are not sufficient to provide all numbers.

You need to fold in the little finger, and hold the other four fingers flat on the left hand, in this case we will have one unit which if we hold it on right hand it will mean one thousand.

Similarly, we fold the second finger or ring finger and keep the next three fingers unfolded on the left hand meaning two units and on the right hand meaning two thousands.

Third finger or middle finger should be folded with two first fingers, hold the others (thumb and index finger) on the left hand meaning units and on the right hand meaning 3000".

The method of calculating by fingers used to be applied in ancient times and had its own terminology, which was being used until the end of the 18th century.

In the fourth chapter operation is stated clearly and concisely. The method of calculating the square root of the numbers is presented. As follows:

"For example, to calculate square root of 11: 9 is the nearest integer to 11 which has square and its square root is 3. We subtract 9 from 11, we obtained 2. We double 3 and achieve 6. Remaining 2 is considered two sixth. Therefore, we find that the root of 11 is three and two sixth, or 3 and one-third".

Artavazd's method can be demonstrated by the following formula:

$$x_1 = \sqrt{A} = \sqrt{a^2 \pm b} \approx a \pm \frac{b}{2a}$$

In the first part of the fifth chapter there is a discussion about order of numbers. Artavazd explains calculation method of addition, subtraction, multiplication, and division in details:

"Simple multiplication is done in this way: About what is related to double and triple and so on, it should be noted that one unique method can be performed so that the subject of the Indian multiplication becomes completely explained. It can be done like this: The first two times, then

twice with a double, then triple, and so on. In the second step, three times of the first, then twice, then three times, and so on, the same operation with four times, five times, and others. "

Finally, at the end of the first work, the multiplication tables are included in which the row of natural numbers up to 10, are multiplied by $\frac{2}{3}$, $\frac{3}{2}$, $\frac{3}{1}$, ..., $\frac{10}{1}$, and the result is written in Egyptian method.

The second part includes a variety of problems. At the beginning he discusses in detail about arithmetic, different chapters and especially problems included in his writing.

In the first part of the work he repeatedly speaks about multiplication, division and square root. Here are some examples:

1- $3 \frac{1}{3} \frac{1}{14} \frac{1}{42}$ multiplied by $3 \frac{1}{3} \frac{1}{14} \frac{1}{42}$ which means calculation of square.

2- Multiply $5 \frac{2}{3} \frac{1}{5} \frac{1}{110} \frac{1}{830}$ by $8 \frac{2}{3} \frac{1}{4} \frac{1}{156}$.

3- $5 \frac{1}{5}$ times $7 \frac{1}{7}$ and the result multiplied by $9 \frac{1}{16} \frac{1}{18}$.

4. Dividing $3 \frac{3}{1} \frac{14}{1} \frac{42}{1}$ by 10.

5. Calculate the square root of 10, 3 and 24.

Nikoghayos Artavazd, by solving specific problems presents general methods for solving the problems. Two examples: To calculate the square of $3 \frac{1}{3} \frac{1}{4} \frac{1}{42}$ writes the fraction in Egyptian style, changed to today's format, showing combined value of $3 \frac{7}{3}$ as $\frac{7}{24} = 4 \frac{3}{7}$. Multiplies $\frac{7}{24}$ by $\frac{7}{24}$ (as we have today) and thereby obtains $11 \frac{49}{37}$.

Square root calculation method is as follows. To calculate the square root of 10 the nearest square of 9 is subtracted from 10 getting 1. Approximately double the square root (result 6). The difference between

10 and 9 is 1, and it is divided by 6 so $1/6$ is obtained. The square root of 10 is approximately $3 \frac{1}{6}$.

$$\sqrt{10} \approx 3.$$

Artavazd finalized his work with 18 problems that are allegorical. To become familiar with them, we shall bring two examples:

Problem 4: A man asks somebody a question. I got a deal done and bought 3 pounds and $1/3$. Then sold and bought it again at $3 \frac{1}{5}$. I discovered that I have gained 10 Nomisma. We want to know how many Nomisma was used in the transaction. That person answered 240.

Problem 5: Someone was asked a question: if you give me six of ten Asarion and I add them up with what I have, I'll be twice higher than you. The man replied, no, you give me six units of your assets and I will have as much as you. Now I am asking, how much does each one of them have?

Solution of the second problem of Artavazd by using modern style is as follows:

1. $1/5x + 1/6x = 21$
2. $1/4x + 1/5x = x - 12$
3. $x + 1/4x + 1/5x = 30$
4. $(3 \frac{1}{3} - 3 \frac{1}{5})x = 3 \frac{1}{5} \cdot 10$
5. $2(y - 6) = x + 6, x - 6 = y + 6$
6. $x + y = 100, 7x = 9y$
7. $103/53 = 1000/x$
8. $y/5 + x = 10000, 1/7 + y = 10000$
9. $x(1 - 1/8 - 1/9)(1 - 1/3 - 1/7)(1 - 1/3 - 1/4)(1 - 1/4 - 1/5) = 1 \frac{1}{2}$
10. $x(1 \frac{1}{2} + 2 \frac{1}{2}) = 7$
11. $2[2(2x - 15) - 15] - 15 = 0$
12. $3x = 5, 3y = 7, 3z = 9$
13. $380 \times 85 = (24 + 85)x$
14. $x + b - a = y - b + a$

$$15. \frac{1}{3x} + \frac{1}{4x} + \frac{1}{5x} + \frac{1}{6x} = x - 36$$

$$16. \frac{1}{3x} + 15x + x = 138$$

$$17. x(1 - \frac{1}{3})(1 - \frac{1}{4})(1 - \frac{1}{5}) = 24$$

$$18. \frac{10}{2} = 40 / x_1, \frac{10}{3} = 40 / x_2, \frac{10}{5} = 40 / x_3$$

Since the methods of Artavazd are so interesting, his name is recorded in the Great Encyclopedia: "The work is very valuable document about how Byzantine computing was. In particular, he follows pure tradition of the Greek."⁴³

Artavazd's mathematical methods are of particular importance in the development of mathematical thoughts and, in particular in arithmetic.

Despite the invasion of Seljuk Turks and the Mongols in the 12-14 centuries, the natural process of development of the Armenian sciences did not stop completely. Still schools and universities such as Glazor and Tatev were active. Glazor university was founded in 1280 and was active for 58 years. Tatev university survived for 70 years until 1415. The major problem of these universities was providing the needs of large and small feudal lordss and the Armenian churches. The number of students was very limited. For example, only 80 students were studying at the University of Tatev and main courses were theology and philosophy.

Graduates of these schools should learn their mathematics and calendar, otherwise they would not be able to calculate important Christian feasts, so the teaching of mathematics, astronomy and the calendar was necessary and for this reason sometimes some of the students devoted themselves to the natural sciences.

⁴³ - La Grand Encyclopedie, XXVIII, Paris, p. 566.

During the domination of kings of Cilicia (1080-1424) the level of culture and science and mathematics had excellent progress and teaching mathematics became one of the fundamental fields.

According to the progress of Armenians in 12-14 centuries, it can be concluded that Artavazd owed a part of his knowledge in the field of computational science to the scientific development in Armenia. As a result of the Armenian move to the west and to the territory of the Byzantine Empire, many of the scientific works, including works of Shirakatsi were transferred to those regions.

One example of this transfer is appearance of mathematician and philosopher Leon in Byzantium who played a significant role in the development of science in that land, especially in Constantinople and its university.

Section 10

Application of decimal numbers in Armenia based on the Armenian sources

Social conditions in the 13-16 centuries

First half of the 13th century coincided with the devastating invasion of Mongols to the Western Asia (Asia Minor, Armenia, Syria, Lebanon, Jordan, Palestine, Iraq, Iran, Saudi Arabia, Yemen). Then in 1236 they captured northeast territories of Armenia and other parts of the country. Horror and terror of Mongol domination lasted more than 150 years.

In the second half of the 12th century. North eastern part of Armenia that was freed by Zakarian brothers with the help of the leaders of Georgia from tyranny of the Seljuk and passed an era of peace and economic progress and development, encountered political and cultural insecurity and chaos by the Mongol invasion. On the other hand the Mongol forces were slaughtering active people during their inroads, and on the other hand by imposing heavy taxes they disrupted the economic life of the country. During Mongol rule many cities and villages, including the famous cities of Ani and Dvin were destroyed.

14-15 centuries were the most tragic period of history of the Armenian people. In the second half of the 14th century after the collapse of the rule of the Mongols in the Near East, this territory suffered from the brutal invasion of Tamerlane, Qaraqoyunlu and Aghkoyunlu tribes and others. As a result, the historical developments, economic and cultural life fell apart and a significant proportion of people left their homes and migrated abroad (Crimea, southern Russia, Poland, Egypt, Syria, Constantinople, Izmir and Mediterranean coastal cities).

In the early 16th century. Two ethnic empires, Ottoman Turkey and Safavid Iran were formed and in order to expand their power and territory started deadly wars against each other which continued about 125 years, and the Armenian territory was their battlefield and it was exchanged between them several times.

It was natural that in such uncertainty in the 15-16 centuries, scientific and cultural centers of Armenian moved to safe areas. However, the Armenians did not stop the progress of the new systems and promoted the development of mathematics.

Usage of the decimal system

Today, all nations use the decimal system. Different numeric systems stood against it but this system showed stiff resistance, and eventually decimal system became dominant on all of them and its use was common among different nations.

Before using the decimal system, alphabetical order was being used. The progress of the counting system is closely related to the evolution and progress of human societies. The emergence and development of logical systems, was reflection of the general trend the social changes. New comprehensive systems gradually replaced the previous methods.

Decimal system that was introduced by the Indians in 5-6 centuries had growth and progress later on. In the same period, zero value entered into the numeric system. Arabs in the 8th century AD got familiar with the Indian methods and principles of computing and mastered it and with the remains of Greek mathematics and science, later on they moved it to Europe. Signs and symbols of numbers changed largely over the time.

By translating mathematical works of illustrious Iranian mathematician Muhammad Khwarizmi in 9th century from Arabic language into Latin, in the 12th century decimal system was developed in Europe.

In the year 1202 AD. The book of Leonardo of Pisa, known as Fibonacci, was released which included algebraic problems and was under the title of “Liber Abaci”. It was very interesting and the author gained great reputation. In this book Indian numbers were used with Arabic signs.

During the centuries, there was a big disagreement and argument between the new and old systems (using the Roman alphabet, etc.) which led to victory for the new system. The argument occurred in different countries in different historical periods. For example, in Germany, France and England until the second half of the fifteenth century Indian numbers were not used.

To find out from when the Armenians started to use the modern numbering system, we need to have a look at the Armenian sources on mathematics.

The contents of the arithmetic textbook of the unknown author

One of the Armenian mathematical resources which has been in center of attention, has been the Armenian manuscript No. 8716 at the National Manuscripts Library of Yerevan, called Matenadaran (in Armenian language), which was discovered by G. B. Petrosian, a renowned Armenian mathematical scientist. This work is a textbook for teaching mathematical techniques based on the decimal system.

This work is of great importance in the study of mathematical ideas among the Armenians and consists of two parts and a total of 42 pages. The first

part consists of introduction and five chapters and a total of 16 pages. In this section, the author tries to explain decimal system and four major arithmetic operations and explain the transition from the old system to the new system. The second part of the book is about solving various problems and different principles of arithmetic.

In the preface, the author of a textbook speaks about arithmetic calculations associated them with the four main elements.

This new system only deals with the letters (number sign) that they can be written and read any type of numeric, while old system used only the 36 letters of the Armenian alphabet. The author writes:

"Arithmetic techniques are like this: First, one must recognize that it is (derived from) the four elements. The letters are the first elements, then the letters come in. But you, the reader should know that this method has only nine letters and not more. Whether one thousand or a few (tens of thousands) it will not have more than of 9 numbers.

And nine numbers are:

ա բ գ դ ե զ լ խ
1 2 3 4 5 6 7 8 9"

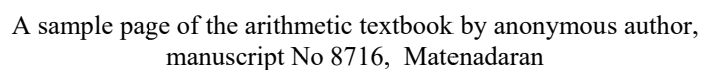
The author writes down the first, second, third and fourth digits with alphabet and compares them with four main elements of nature and calls the new decimal system as "shining and bright" and the earlier system as "dark and gloomy".

In the preface he states that during his travels to various countries He had got familiar with the new system of counting. He started that at the beginning he did pay attention to the importance of the new system, but later realized the importance and decided to study it thoroughly and spread it among the Armenians. In this regard he writes:

"I have studied this matter. I ignored it because I was unaware of all these things and good things I saw, for some time I missed the happiness of the universe, so I asked the person for help who provided me with the book for free and I tried to master computational techniques which was hard for me to understand. But what I was unaware, they were understandable for Italians, Romans and other nations".

The author then reminds that although many people wrote and read different counting system with new signs (numbers), but the evidence shows that they used a unique system. The author writes in this respect:

"But be aware that this technology is far different from another because it only counts the numbers, and deals with very large numbers and in a blink gives very large numbers in the language of Tajiks (Arabic) it is called "Raqam"(digit). Generally there are only nine signs and this is the way they are written in Armenian, French and Arabic.



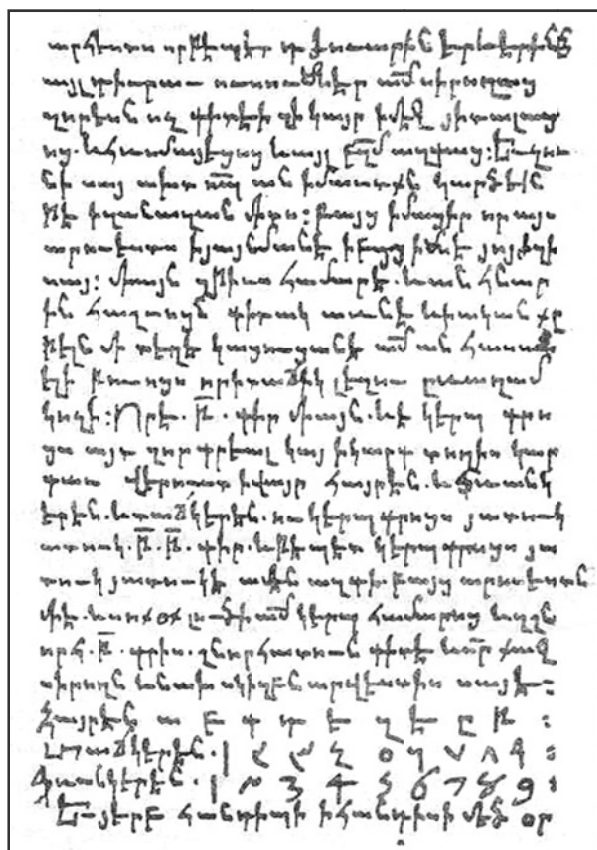
A sample page of the arithmetic textbook by anonymous author,
manuscript No 8716, Matenadaran

Although the method of by different nations are unique, but a there is only one technique, by which everything is solved and the experts and lovers of it know very well. Start of this technique is as follows:

Armenian: ա բ գ դ ե զ լ թ

Arabic: ١ ٢ ٣ ٤ ٥ ٦ ٧ ٨ ٩

French: 1 2 3 4 5 6 7 8 9



Another page of the arithmetic textbook by anonymous author,
manuscript No 8716, Matenadaran

The fact that author considers the new system better than the previous one is obvious at the following lines:

In the first chapter of the first part of the book, the author describes the computational technique of 9 signs. To illustrate the calculation method he used letters of the Armenian alphabet, by which he could show all numbers.

The unknown author by definition of computational method shows that this technique focuses on five problems: first, order of letters and their numerical values, second, addition of numbers, third, subtraction, fourth, multiplication, fifth, division of numbers. By knowing these techniques we can perform any arithmetic calculation. The author then presents the definition and explanation of each of the five operations discussed above.

In the chapter "numerical order, the author states that there are four orders: The first, unit, the second tens, third hundreds, fourth thousands.

Thus, from *u* (the first letter of the alphabet) to *թ* (ninth letter of the alphabet) are the first digits or units. The letter *ժ* (tenth alphabet letter for ten) for the first decimal value, *ի* (eleventh letter of the alphabet to the value of 20) for the second decimal value and so on.

The numbers in the third place *Ճ* represents hundreds, *ւ* for two hundred, *Յ* for three hundred and so on.

Fourth digit presents thousands and if we do not deal with *Ա* as in *.Ա.Ռ.* (*Ա* to determine an Armenian word and *Ռ* as 1,000 for a thousand) *.Ռ.* is for one thousand and *.Բ.Ռ.* means two thousands and so on.

The author then regularly speaks about the unit, the tens, hundreds and then incorporates the thousands, described one category to the next level in the decimal system and how to read and write number categories:

"When we go to the second level, the first tens is obtained, then second tens, then third and so on until the ninth tens which is Ղ. (In the Armenian alphabet has a numerical value of the ninety), which is nine times ten, then we get to the next category, we find that there are hundreds which are ten times ten:

Two hundred, three hundred, and a hundred and a thousand in the next category, which is 10 times 100 and continues up to 9 thousand and then the fifth is 10,000. That ends and we find that 9 times 10,000 is 90,000 and so on. The higher order is 12, from 100000000000 to 900000000000. Thirteenth order begins with 1000000 millions and the numbers continue in this way”.

In the textbook Armenian alphabetic values are used as the following table:

Tens	Hundreds	Thousands	Tens of thousands
Ժ- 10	Ճ-100	Ռ- 1000	.Ա.- 10000
Ի - 20	ԲՃ-200	ԲՌ-2000	.Բ. - 20000
Լ - 30	ԳՃ-300	ԳՌ-3000	.Գ. - 30000
Խ - 40	ԴՃ-400	ԴՌ-4000	.Դ. - 40000
Ծ - 50	ԵՃ-500	ԵՌ-5000	.Ե. - 50000
Վ - 60	ԶՃ-600	ԶՌ-6000	.Զ. -60000
Հ - 70	ԷՃ-700	ԷՌ-7000	.Է. -70000
Ձ - 80	ԸՃ-800	ԸՌ-8000	.Ը. -80000
Ղ - 90	ԹՃ-900	ԹՌ-9000	.Թ. -90000

The author, then explainu the numbers in the table. Then talks about the four main operations and presents some examples. He performs addition operation in the same way that we practice today. However, he also provides a test and verification. An example with step by step operations:

$$\begin{array}{r}
 86454 \quad 7 \\
 33593 \quad 7 \\
 65432 \\
 \hline
 185479
 \end{array}$$

Starting from the left, we first add 8 and 6 giving 14 in accordance with the method of the author 9 is subtracted, result is 5 and 5 is added to the next number which is 4 giving 9 then subtract 9 the result is zero. Then the next zero added to 5 and 4 together, we get 9 and subtract 9 from that amount, the result is zero. The second number, the sum of 3, 3, 5 is 11, subtract 9, the remaining 2. 2 is added to 3 gives 5 which is added to first digit of the third number which is 6 result is 11 minus 9 giving 2. This 2 is added to next digit 5 giving 7 which is less than 9 so is added to next digit 4 sum is 11 and minus 9 results 2 added to next 2 and 3 giving 7.

If you do the same with figures of addition result we will get 7. Thus, by applying this method we obtain 7 both for tree numbers and their addition resulting the same 7, so this addition operation is correct.

Anonymous author performs subtraction the same way we do today. In order to test the validity of this operation he adds minuend to remainder and if subtrahend is obtained so subtraction has been done correctly or if deduct the amount of remainder from subtrahend, minuend is obtained so subtraction is done correctly.

In the textbook there are several examples for multiplication and how to write multiplicand and multiplier and the product in the way we are doing today. However, the details of multiplication have not been specified. To test the accuracy of multiplication anonymous author presents the following:

We add up the digits of multiplicand then subtract 9 until we get a number lower than 9, and write it down. Then we do the same with multiplier. We multiply these two numbers and deduct 9 values and write down the result which is less than 9. We do the same with the product. If these two numbers are equal then multiplication operation has been down correctly. Note the following example:

$$\begin{array}{r}
 4607 \\
 \underline{254} \\
 18428 \\
 23035 \\
 \underline{9214} \\
 1170178
 \end{array}$$

$$4+6+0+7=17-9 \rightarrow 8$$

$$2+5+4=11-9 \rightarrow 2$$

$$8 \times 2 = 16 - 9 \rightarrow 7$$

$$1+1+7+0+1+7+8=25-9-9=7$$

7=7 So the operation is correct.

The author of the textbook performs division essentially in a manner that the Italians called it "Galra", because after division, numbers are like a ship of the same name. According to this method, the divisor is written under dividend (starting from left). If the first digit of divisor is smaller than the first digit of dividend, in this case the first digit of divisor is written under the first digit of dividend. Then the second under second, third under third, and so on. The part of dividend that is over the first digit of divisor, will be divided to the last number and the result (quotient) will be placed on the right side of small bracket which is put after the first digit of the dividend. The first digit of quotient will be divided to first digit of

To show this operation we present another example from the textbook:

Example: calculate 25439 divided by 17:

Step 1: We write down two numbers.

Step 2: Divide 25 by 17 divide. We write quotient on the right side of the bracket and the remaining on top of 5 and we delete 25 and.

1) 25439
17

Step 3: We move divisor one digit to the right, so that the first digit of the divisor falls under the second digit of divisor and the second digit is shifted to the right. 84 (8 and the second digit after 25) divided by 17, result is 4 and the remaining is 6. We write 6 beside 8 and cross out 8, 4, 1 and 7:

1
86
25439 (14
277
1

Step 4: We write divisor in a way that one digit moves to the right, and the first digit falls again under the second digit of divisor and the second digit of divisor is shifted to the right. The next figure in dividend is 3 which is beside remainder and 163 is divided by 17 so quotient is 9 and remainder is 10. We write down so that 1 goes on top of 6 and 0 over 3. We cross out 1,6,3,7 (because they have participated in these calculations).

11
860
25439 (149
1777
11

Step Five: We write down divisor and we move one digit to the left (as in the third and fourth step was done). 109 (number 9 with the remaining 10 will be the next dividend) is divided by 17, with the quotient of 6 and remainder is 7. We write 7 on 9 of relevant dividend and delete one and zero, 9 and 7:

11
8607
25439 (1496
17777
111

To test the correctness of the Division we do the following: We add up the digits of dividend and deduct 9 by 9, we do the same with divisor and quotient:

25439 \rightarrow 5 (a)
17 \rightarrow 8
1496 \rightarrow 2
8*2=16 \rightarrow 7
Remainder=7
7+7=14 \rightarrow 5 (b)
(a)=(b)=5 So division is correct.

In the second part of the textbook of anonymous author different problems and some relevant rules are discussed. Now we present Problem No. 1 and its solution:

"There were two friends. The first one had 167 and second 243 Sarmya. They gained 134 profit out of their business. How much is the share for each one? As it is written $243/79$ and $167/54$, 134 is earned. "

The author solves this problem this way:

$$243 + 167 = 410$$

$$167 \times 134 = 22378$$

$$22378 / 410 = 54$$

$$410 / 167 \times 134 / 1$$

$$134$$

$$\underline{167}$$

$$22378$$

$$23$$

$$1878$$

$$54) 22378$$

$$4100$$

$$41$$

To find the share for the second person:

$$243 \times 134 = 32562$$

$$32562 / 410 = 79$$

$$134 \times$$

$$\underline{243}$$

$$32562$$

$$17$$

$$3862$$

$$32562 (79$$

$$4100$$

$$41$$

Problems No. 1, 2, 3, 4, 5, 6, 7 and 8 are of the same kind.

Now it is necessary to solve the problem number 9:

"Someone had 20 Qrush money. He lent it for a period of 8 months for 12 Qrush. But his friend, who had 30 and 5 Qrush says that the loan was for 3 months. How much is the interest for 35 Qrush? Look at the following formula: $(20-8)/(35-3) \times 12/1$ “.

The author solves this problem:

$$12/8 \times 20$$

Then calculates how much profit is for 35 Qrush in a month. For this purpose he multiplies $12/8 \times 20$ by 35:

$$12 \times 35 / 8 \times 20$$

Then he calculates the profit of 35 Qrush in three months and multiplies latter amount by 3:

$$12 \times 35 \times 3 / 8 \times 20 = 7$$

Result is 7.

The author solves the problems No. 1 to 9 by four main operations and integer numbers. From the tenth on, the concept of fractions and decimal numbers are raised. Now we present problem number 10:

"(10) Here the concept of quarter and half is discussed. One out of the two is called half, quarter is one out of four. For example, "When I say one and a half or two and a half or three and a half or so and say one and a quarter, two and a quarter, three and a quarter, again, one and three-quarters, two and three fourth and so on. In this case, you can do anything you want, ie addition, subtraction, multiplication and division".

Then the following fraction amounts are presented below:

$$1 \frac{1}{2} \quad 4 \frac{1}{2} \quad 7 \frac{3}{4}$$

$$2 \frac{1}{2} \quad 5 \frac{1}{4} \quad 8 \frac{3}{4}$$

$$3 \frac{1}{2} \quad 6 \frac{1}{4} \quad 9 \frac{3}{4}$$

The example:

"Add 37 Dram and 3 Dang (Dang is equal to a quarter of a Dram) to 16 dram and 3 dang. We want to know how much is result. Using formulas please find the answer. "

$$151/4 \times 67/4 = 54 \frac{1}{2}$$

However, the fraction $1/4$ is used but the denominator is 16.

The author presents the concept of arithmetic progression in the following question.

"A person hired a worker for 24 days and told him he would pay 1 Stak for the first day, 2 Staks for the second day, 3 Staks for the third day, 4 Staks for the fourth day and one more Stak for each more day until the last day. And worker stayed 24 days. How many Staks did was he paid?"

To solve the problem he adds first sentence which is 1 to the last sentence 24, then multiplies the result by half of 24, result is 300.

Which is formulated as our times:

$$S_n = \frac{n}{2}(a_1 + a_n).$$

The operation on fractions is as presented in the following example: For example, "400 divided by $5/7$ is as follows:

$$\begin{array}{r} 400 \\ 7 \overline{) 50} \\ \dots \\ 2000 \\ \underline{2800} \\ 3000 \overline{) 00} \end{array}$$

No multiplication of 400 by 4 and one-quarter:

$$\begin{array}{r} 400 \\ 4 \overline{) 25} \\ \underline{2000} \\ 800 \\ \underline{1600} \\ 1700 \overline{) 00} \end{array}$$

At the end of the book there are discussions about dividing a number in half.

When was the arithmetic textbook written?

The National Library of manuscripts, Matenadaran in Yerevan has manuscript No. 8716 related to the 18th century. A calendar is attached to the end of this version of the textbook which is titled "Commentary on the calendar by Rev. Samuel at the request of Kalim Stephanos" the author of which lived in the 12th century.

The arithmetic textbook has several tables of polygon numbers. This and a comparison of numerical digits and signs used in the textbook with various resources⁴⁴ help us to consider that the book is written probably in the 15th century or the beginning of the 16th century.

⁴⁴ - Florian Cajori, A History of the Mathematics, Edessa, 1917, p. 154.

Section 11

Armenian mathematical resources (17-18 centuries)

Armenians had hard political times during 17-18 centuries. In the unstable political and social conditions, western part of Armenia known as Western Armenia was surrendered to the Ottoman Turkey in accordance with the 1639 Convention between Safavids of Iran and Ottomans and the Eastern Armenia still continued to remain under the domination of Iran. However, the Armenian territory served as a battle field between Iran and Ottoman army, so it led to the social and economic conditions and political instability of the region. However, the Armenians continued to develop and promote science and culture, language and art.

In these centuries, scientific and cultural activities of the Armenians were not limited to the mainland Armenia. Armenians of Diaspora (as a result of instability in the mainland they had gradually migrated to countries near and far) in turn shared the progress of science and culture of Armenia.

In 1512. Hakop Meghapart in Venice, Italy, published the first Armenian printed book. In the 17th century Armenian printing house in Lvov, New Julfa, Isfahan (1636, the first printing in Iran), Amsterdam and other cities were established.

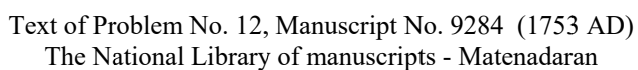
In the late 17th century and early 18th century Armenian schools were established in various colonies and many activities were done to promote science and culture among the Armenians.

To judge about the state of mathematical knowledge among Armenian we need to refer to the manuscripts and printed works of this period.

Handwritten mathematical works

In the 18-17 centuries, special attention was paid to the mathematical literature of Russia.

Armenians during their trades with Russia got familiar with their math. Carpet, silk and other goods produced in Armenia were exported to Russia. An Arab traveler in the tenth century reports about existence of Armenian carpets in Tsar's tents. Many Armenians in the 12th century lived in the city of Kiev. Armenian business in Russia from the 15th century became more organized and steady and they were influential in many cities of Russia. Armenian trade relations with Russia, particularly after the conclusion of a trade agreement between the New Julfa Armenian trading company with Alex Mikhailovich Tsar of Russia in 1667, entered their trade into a more serious stage.



If until the 17th century the mathematical manuscripts were less practical and more theoretical, but in the works of the 18th century we see the details of the application of mathematics and weights and measures used in Iran and Russia.

In the manuscript number 8424 written in 1744 in Astrakhan (north of the Caspian Sea, off the coast of Russia) there are ten arithmetic problems (pages 43A to 45A) that include the terms like pounds, ounces, the ruble.

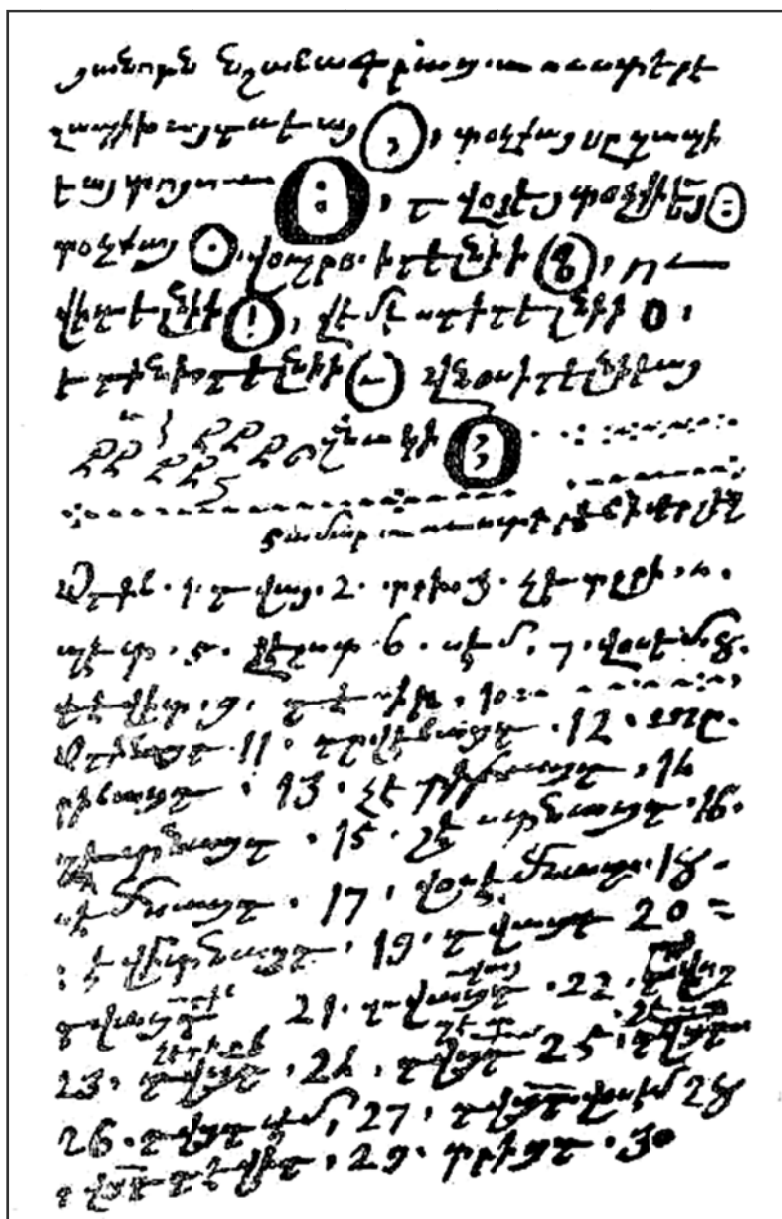
Manuscript No. 9284 (written in 1753 AD.) that consists of 23 sheets, is dedicated entirely to the techniques of computation and includes 20 problems and the Weights and Measures of Iran and Russia (rubles, kopecks etc.). Problem 12 has such start:

"Again, my brother, to deliver a promissory note to Amsterdam, they are asking money to give us some Gilders which equals 365 rubles and 55 kopeks".

In manuscripts No. 8740 written in the late 18th century there are various Russian mathematical and grammatical terms.

In the 8540 version, dated 1807 (pages 59 a -58 b) there are numbers with Russian names. In a page from the manuscript, entitled "Russian numbers" numbers one to hundred are mentioned in the form of the Armenian letters.

Manuscript No. 9627 written in 1830 is probably copied from an older text contains 100 pages and is totally prepared for business operations. The manuscript belonged to Stepanos Boghosian Serbrakiants. In the problems raised in this text, the relation between Iranian and Russian weights and values are discussed, such as 7 Iranian Gaz equals 10 Russian Gaz, 100 Iranian Methghal equals 108 Russian Methghal, and so on.



Manuscript No. 8740, includes Russia grammatical and numbering terms and teaches numbers from 1 to 100 with Armenian letters

A book titled "The Notebook" by Avetic Tigranagertsi is one of the most interesting works in his time. Avetic Tigranagertsi (Avetic Amedetsi or Avetic Baghdasarian. Year of birth and his death are not known to us) was one of the most outstanding cultural and scientific figures of the 17-18 centuries. He was expert in mathematics, astronomy, calendars, history and writing in different languages such as Arabic, Persian, Assyrian, Georgian, Hebrew, Coptic, Turkish, Greek, Latin and French. As its name implies, he was born in the capital of the kings of Armenian Artashesian dynasty, Tigranagert in Western Armenia, and lived, studied and performed scientific researches in different cities such as Erzurum, Van, Baghesh, Sebastia and Tokhat. In order to continue his education he spent some time in Iran and Arabia. Avetic paid special attention to the natural sciences, geography and historiography and due to the enormous interest in science he was faced with hatred and jealousy of the reactionary. By the help of the Armenian and foreign sources wrote some works in astronomy, chronology, geography and music. As previously mentioned, the manuscript of his book entitled "The Notebook" has come to us, his other works include "Astrlab" which he wrote in the years 1709- 1695, in which he explains about various astronomical instruments. He wrote "Voskeporic" in 1700, "Horoscope book," in 1719, "Quasars" and "World view".

Tigranagertsi by using the Armenian and foreign sources, in the years 1712-1701 wrote a very important chronicle entitled "Aghyusapatmagir" ("Tables of historiography"), including world events since the creation of man until 1714.

Համարացայց:	
1 օրիշ.	31 Դժնյա. օրիշ.
2 Դնյ.	32 Դժնյա. Կնյ.
3 Դն.	33 Դժնյա. Դն.
4 Զնիշ.	34 Դժնյա. Զնիշ.
5 Զն.	35 Դժնյա. Զն.
6 Զնյ.	36 Դժնյա. Զնյ.
7 Սն.	37 Դժնյա. Սն.
8 Սնիշ.	38 Դժնյա. Սնիշ.
9 Դնիշ.	39 Դժնյա. Դնիշ.
10 Դնիշ.	Դնիշ.
11 օրիշնյա.	40 Սոսիշ.
12 Դնիշնյա.	41 Սոսիշ. օրիշ.
13 Դնիշնյա.	42 Սոսիշ. Դնյ.
14 Զնիշնյա.	43 Սոսիշ. Դն.
15 Զնիշնյա.	44 Սոսիշ. Զնիշ.
16 Զնիշնյա.	45 Սոսիշ. Զնյ.
17 Սնիշնյա.	46 Սոսիշ. Զնյ.
18 Սնիշնյա.	47 Սոսիշ. Սն.
19 Դնիշնյա.	48 Սոսիշ. Սնիշ.
20 Դնյա.	49 Սոսիշ. Դնիշ.
21 Դնյա. օրիշ.	50 Դնյա. Դնիշնյա.
22 Դնյա. Դն.	51 Դնյա. Դնիշնյա. օրիշ.
23 Դնյա. Դն.	52 Դնյա. Դնիշնյա. Դնյ.
24 Դնյա. Դնիշ.	53 Դնյա. Դնիշնյա. Դն.
25 Դնյա. Դնիշ.	54 Դնյա. Դնիշնյա. Զնիշ.
26 Դնյա. Զնյ.	55 Դնյա. Դնիշնյա. Զնիշնյա.
27 Դնյա. Սն.	56 Դնյա. Դնիշնյա. Զնյ.
28 Դնյա. Սնիշ.	57 Դնյա. Դնիշնյա. Սն.
29 Դնյա. Դնիշ.	58 Դնյա. Դնիշնյա. Սնիշ.
30 Դնյա.	

Another page of manuscript No. 8540 - Russian numbers 1 to 100 – in Armenian letters

The majority of manuscripts of Avetic Tigranagerdtsi are being kept in the library of manuscripts of Armenian Mkhitarian community on the island of St. Lazare in Venice, Italy. Three manuscripts of his chronology (Manuscript No. 1492, 7424, 6243) at the National Library of manuscripts, Matenadaran. The 1492 version belongs to his own pen.

Original copy of "The Notebook," is not available to us, but we have some information based on a research published in Bazmavb Gazette (Venice). The book is dedicated to astronomy, calendar, music, geography and other sciences and a variety of different tools, such as Astrlab. Avetic has written this book in the late 17th and early 18th century⁴⁵.

The manuscript of "Applications of geometry"

A valuable scientific work entitled "The applications of geometry" is attached to the Armenian manuscript version of "Euclid", but its author is unknown to us.

In the medieval and ancient times the Armenians needed scientific and geometrical techniques to carry out their daily livelihood and rural activities. The manuscript contains 45 sheets of text is written in a single column (on paper size $8/25 \times 6/25$ cm). The manuscript does not include pages of notes and observations, writers name and writing time. The title is "Applications of geometry" or "Practical geometry".

This version consists of 15 chapters and each chapter entitled "The first Application", "The second application", "The third application", etc. On four pages there are 30 shapes with Armenian letters. As we know, in the 13-17 centuries in Western Europe, in order to use geometry in everyday life and use it in surveying, architecture and military techniques, in addition to theoretical geometry, practical geometry was created.

⁴⁵ - Ghevond Alishan, Bazmavep, No. 7-10, 1897.

- Great Encyclopedia of Soviet Armenia, Vol. 1, Yerevan 1974.

Armenian author had exactly the purpose to provide practical geometry and geometric applications. In 16-17 centuries, European mathematicians have written books with the same theme.

Armenian author first provides geometric shapes and how to map and measure them then explains their usage in farms and fields. Now, a brief description about these applications:

- Applications 1 to 6: draw a straight line between points, draw vertical and parallel lines, angles and arcs divided into two equal parts. To do so, use the Pythagorean theorem.
- Applications 7 to 9: Measuring the circle. Provide Astrlab, measure angles by it, a table that was used in 17-16 centuries. With this table and lining different angles can be measured. This table has been prepared on the basis of an isosceles triangle. Each side is 30 steps and the base changes with two fingers (each step equals 12 fingers).
- Applications 10 to 14. Drawing angle, triangle, parallelogram.
- Applications 15 to 17. The method of mapping the accessible and inaccessible areas.
- Applications 18 to 22. Draw equilateral triangle in a circle, square and polygon, drawing the line, draw a circle on three points.
- Applications 23 to 26. About drawing an ellipse, parabola, hyperbola. Explanation of conic sections.

This study shows that the Armenian version of the text is not complete and includes only plane geometry and solid geometry is missing. By examining the version of "Euclid" we can find out that both copies were written by the same copier. Since the mathematical terminology and the Eastern and European terms used in two versions have a similar structure,

therefore it can be concluded that the authors of the Armenian versions must be one person.

Author of the manuscript has used various sources, including an algebra that we are not familiar with it. Comparing the "Applications of geometry" with the works of 16-17 centuries shows that this work must have been written in the 17th century. Armenian manuscripts of Euclidean geometry, as far as we know, belong to Grigor Kesaratsi (Armenian author of the first half of the 17th century) and since the "Application of geometry" is very similar to this work, so it can be assumed that the recent work also belongs to Grigor.

Interestingly, it was common that some expressions were written in European, Arabic and Persian languages. A few examples:

Arabic: algebra

(Greek): parabol

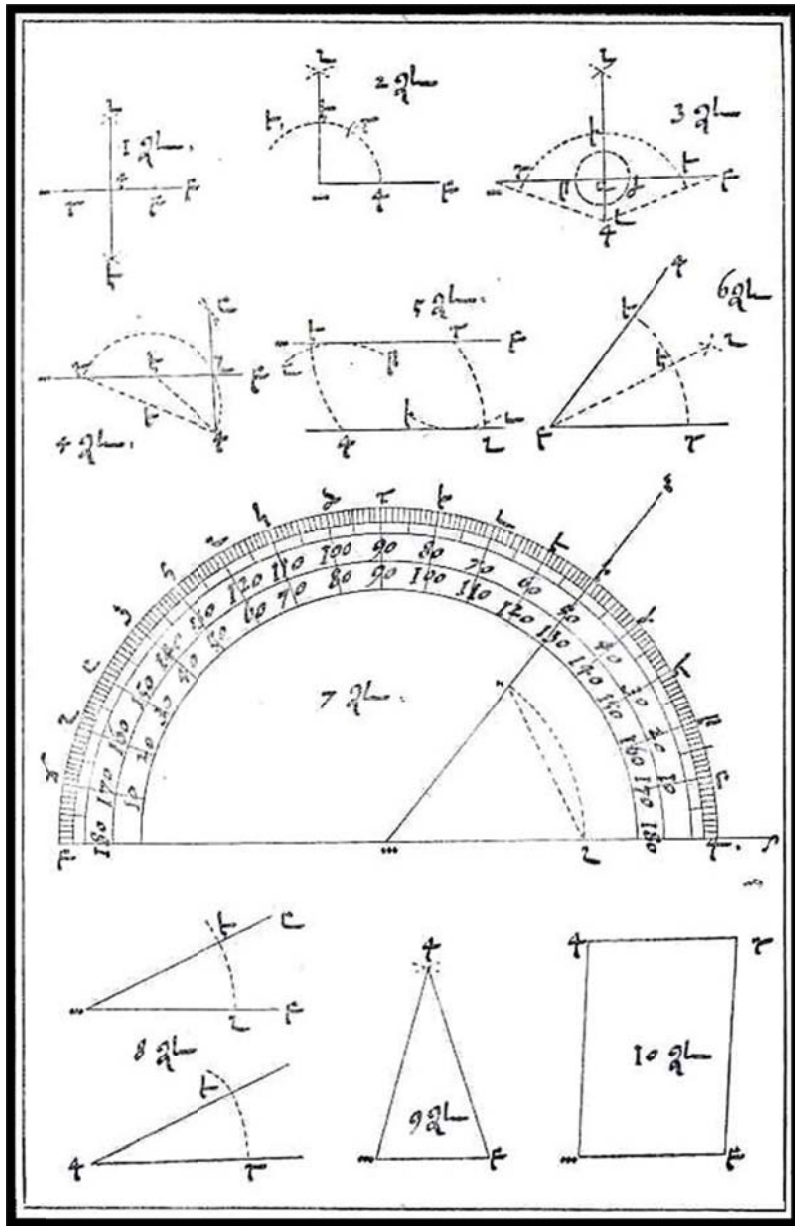
(Greece): ellipse

(In Arabic): merkez

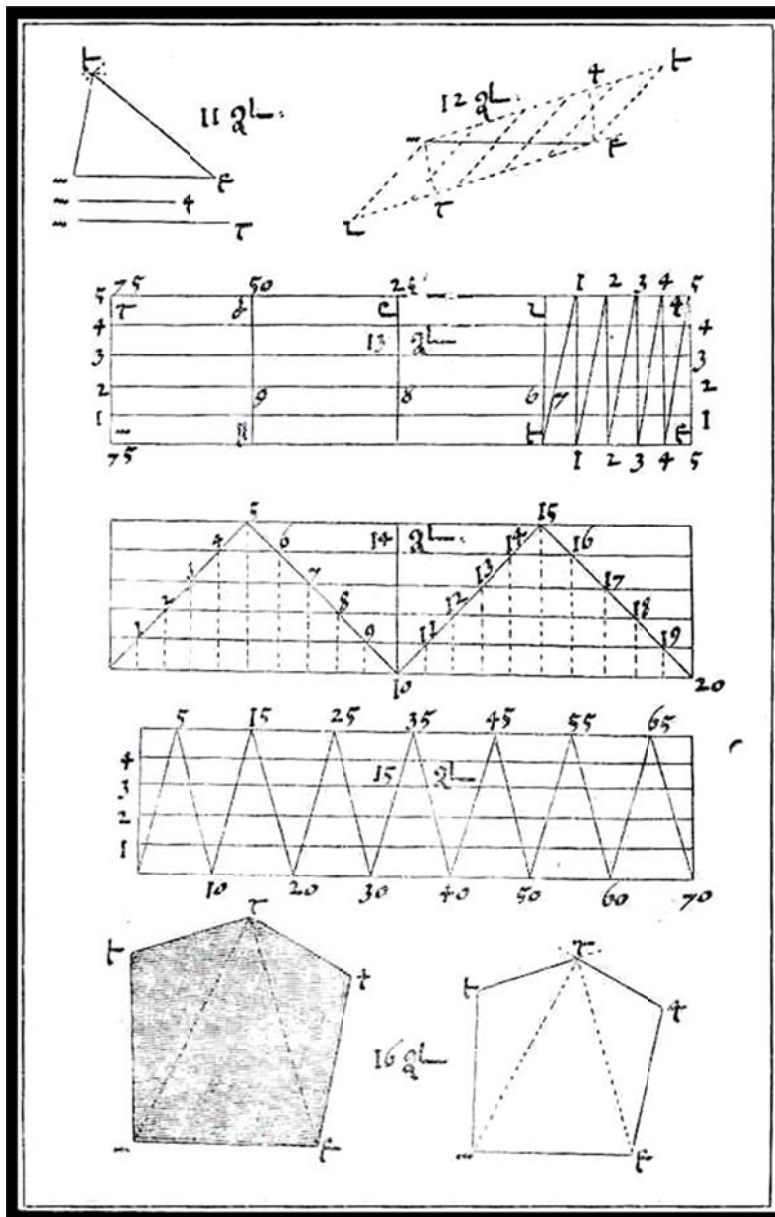
(Persian): cherek

(Persian): pargar

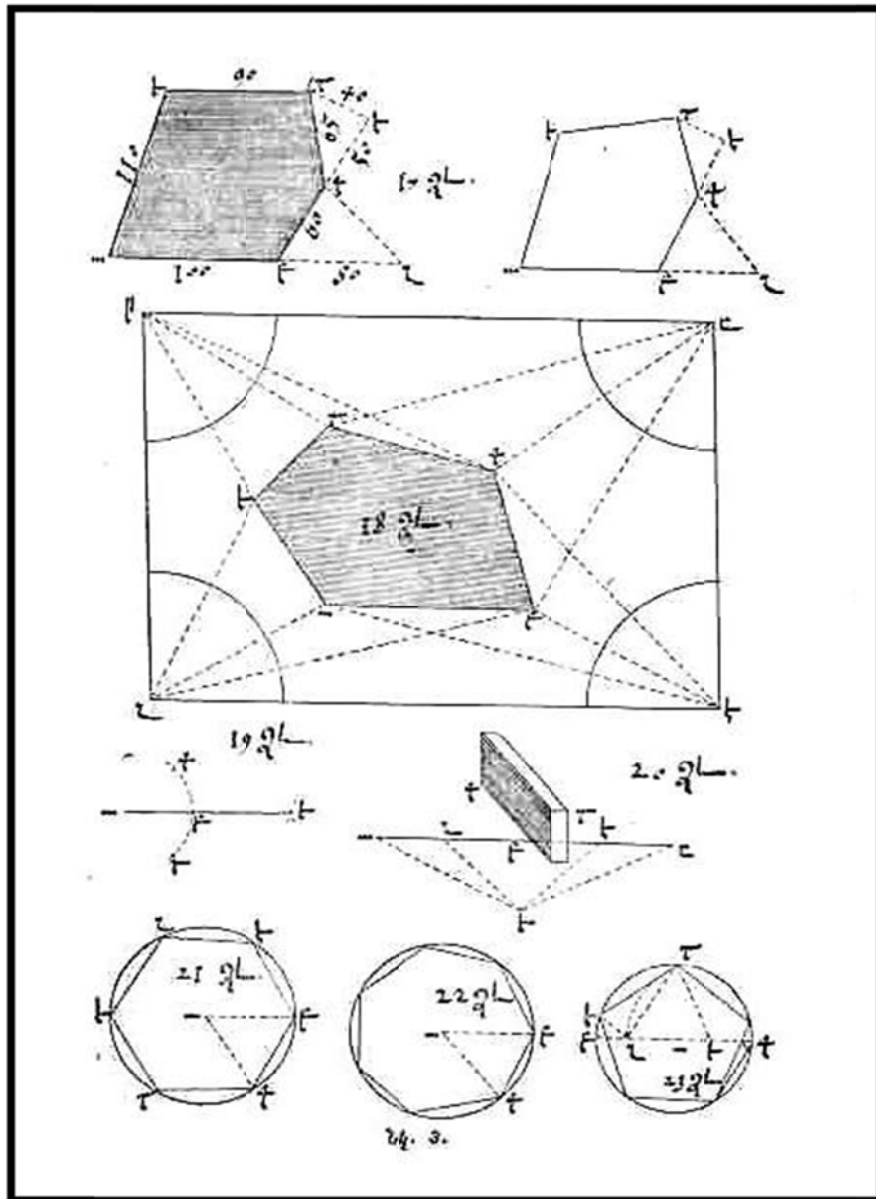
(France): foyer



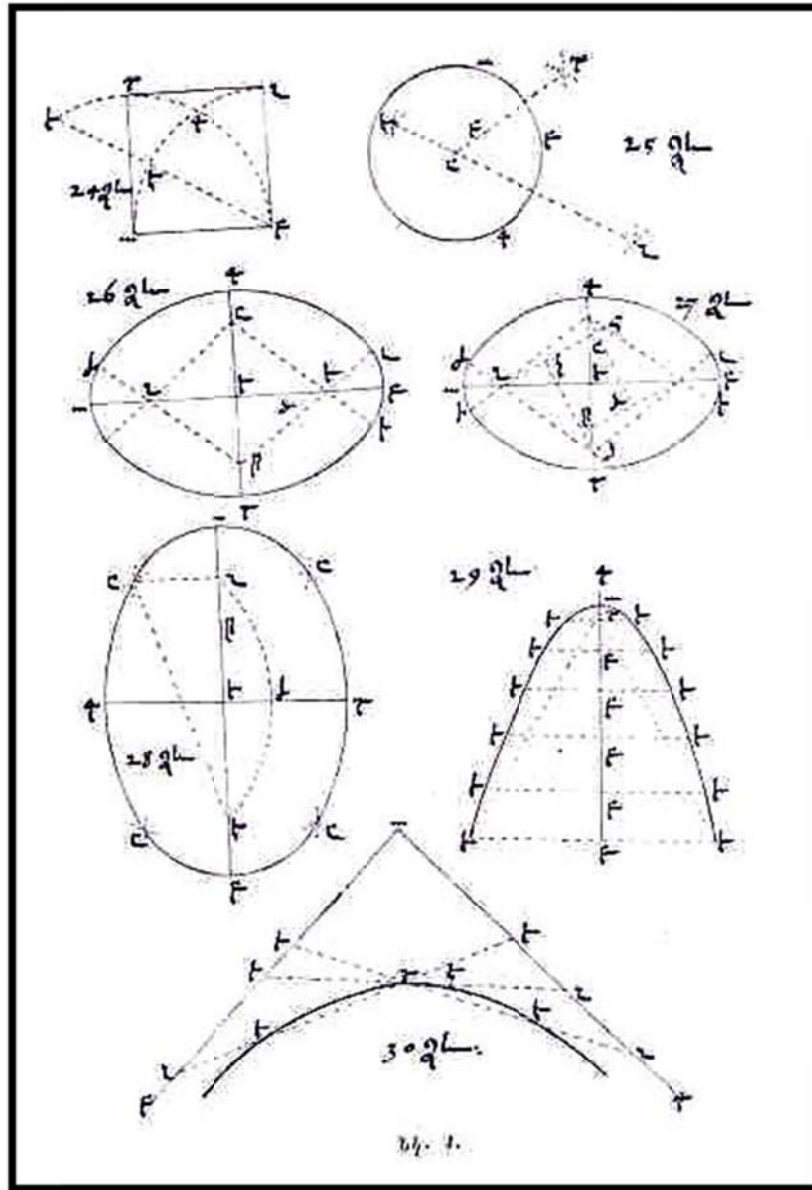
A sample drawing from "Applications of geometry"



Another page of the "Application of geometry" –
depicting various shapes marked with Armenian letters



Another page of "application of geometry"



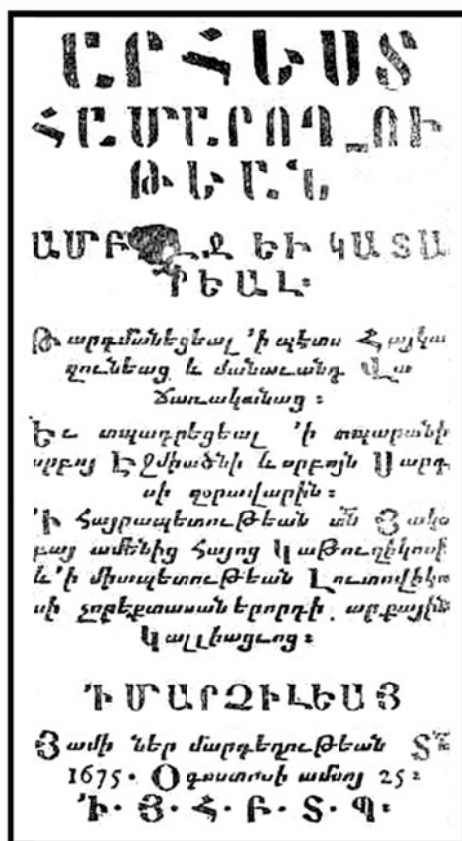
Another page of "application of geometry"

Printed Mathematical books in Armenian

The historical-philosophical research works related to the 18-17 centuries include also some works in geometry and arithmetic. A few works printed exclusively in mathematics are presented here:

1- The first printed book

The first Armenian printed math book is entitled "Techniques of counting" or "Techniques of arithmetic" which was published in 1675 in Marseille, France. Its author is unknown.



The title page of the first printed Armenian book in arithmetic

In the preface of the book reads:

"Based on the needs of the Armenian merchants, who were not familiar with computational techniques, we found it appropriate to translate this book to our mother tongue, so that experts can easily do their work and have ease of mind. Because this book uses integer and fractional numbers with simple examples, as well as their rules, corporate laws, conversion of currency, trade in various commodities and other details. Particularly it is written in spoken language of today, so there is no excuse for lazy people and there are no other complaints and if once they read it carefully, all problems will be solved for them".

The book consists of two parts and 147 pages. The first part contains 5 chapters and 79 pages and the second part contains three chapters. It is noteworthy that the anonymous author of the book has used numerical symbols that we use today, but the signs of addition, subtraction, multiplication and division, and other mathematical symbols are not used.

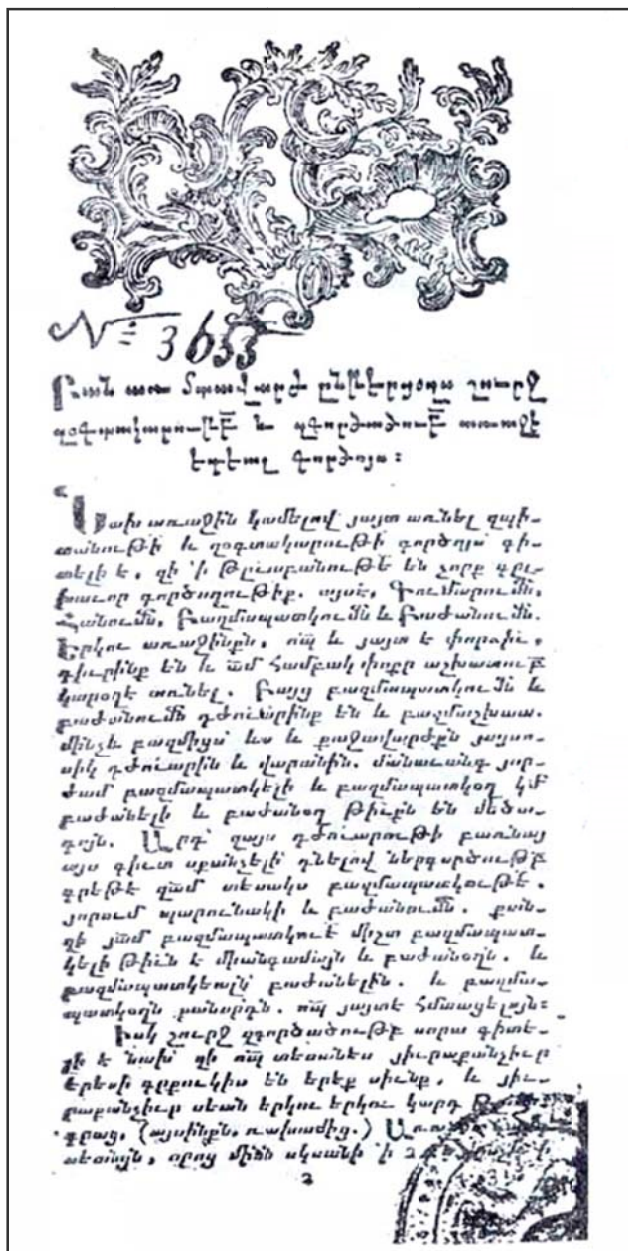
In the first chapter of the book he describes the decimal system in details and uses the term "zero" and calls the numbers as letters. The term "Bivar" meaning 10,000 applies to the million. In the second chapter the basics of addition, in the third chapter subtraction and in the fourth chapter the current method of multiplying numbers are explained.

The fifth chapter entitled "How to divide integers" and the below table is presented before explaining the division:

1	2	3	4	5	6	7	8	9	10	11	12
2	2	3	4	5	6	7	8	9	10	11	12
	4	6	8	10	12	14	16	18	20	22	24
3	3	4	5	6	7	8	9	10	11	12	
	9	12	15	18	21	24	27	30	33	36	
4	4	5	6	7	8	9	10	11	12		
	16	20	24	28	32	36	40	44	48		
5	5	6	7	8	9	10	11	12			
	25	30	35	40	45	50	55	60			
6	6	7	8	9	10	11	12				
	36	42	48	54	60	66	72				
7	7	8	9	10	11	12					
	49	56	63	70	77	84					
8	8	9	10	11	12						
	64	72	80	88	96						
9	9	10	11	12							
	81	90	99	108							
10	10	11	12								
	100	110	120								
11	11	12									
	121	132									
12	12										
	144										

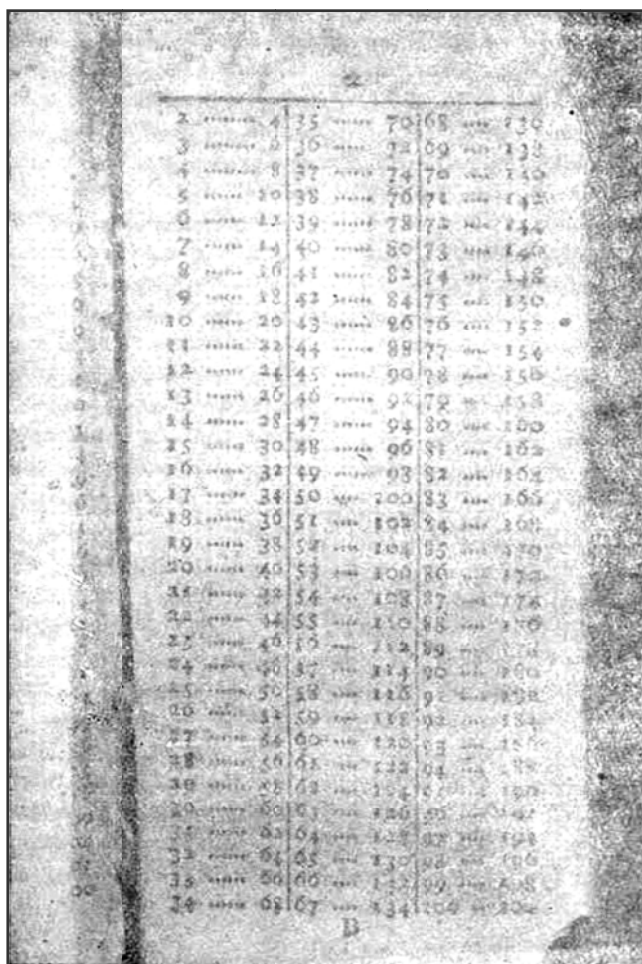
Correctness of the division operation is performed by using numbers 9, 7 and multiplication.

The first chapter of the second part is allocated to decimals and fractions and decimal numbers are used for the weight of gold and such things. Then four main operations are described on these numbers. Before providing details about division there is a detailed discussion about Weights and Measures. Chapters III and IV of the second part is devoted to solving arithmetic problems. The simple and complex proportions and percentages are described.



One page of the preface of the book of arithmetic tables

It seems, from the content of the book, that it is a textbook to help practical business and aims to assist the Armenian merchants in their calculation of their jobs. Some terms in Italian, French, Turkish and the Persian term "Sarmayeh (capital)" are used in the text book. Mathematical theorems and their proofs are not included in this book, and the only practical methods of mathematics are presented.



The image shows a page from an old Armenian arithmetic book. The page is filled with a grid of numbers written in Armenian script. The numbers are arranged in rows and columns, with some numbers appearing to be in different currencies or units, as indicated by the varying lengths and symbols. The page is aged and has a slightly mottled appearance. The numbers are written in a clear, printed font, typical of early printed books. The page is numbered '152' at the top center. The text is in Armenian, and the numbers are arranged in a way that suggests a table of multiplication or division. The page is titled 'A sample page of the first Armenian printed arithmetic tables'.

A sample page of the first Armenian printed arithmetic tables

2- The second printed book:

In the National Library of manuscripts, Matenadaran in Yerevan there is a small book of techniques of arithmetic that its first page is missing, this book consists of 120 pages, which 109 pages are tables. On the first page of the tables the natural numbers from 1 to 100, and four times of them are mentioned, the second page includes natural numbers 1 to 100 multiplied by 2, the third page multiplied by 3 and same in the following pages continues until 100. The natural numbers are multiplied up to 100 by 100, 200 and so on up to 1000. Each page consists of three columns and at the start of each page the details of how to use the table and examples have also been added. With regard to the content and language of writing it can be attributed to the 17th century.

3- The third book:

The third mathematical printed book in Armenian language is entitled "New training methods for arithmetic" published in Venice in 1711:

"New training methods for arithmetic.

The correct method of calculation, the time of trade and purchase, the amount of weight with different prices, for all over the world. For items divided into several sections, to share in the company and eventually mental calculation. For comparison between "Weights and Measures" in different regions of the world.

In 1711 the city of Venice to the great commandment in the (press) Antonius Portoli.

From the beginning to the end it is only one table. At the end of each, there is an advice, in order to use the previous table, then an express post, from here to anywhere around the world."

4- The fourth Book:

The fourth book, is entitled "Arithmetic" by Sukias Vartapet Aghamaliants, printed in Venice in 1781, in 511 pages.

The preface contains 4 pages and the author discusses the importance of arithmetic and some important tips.

Aghamaliants by discussing the Armenian mathematics and writes:

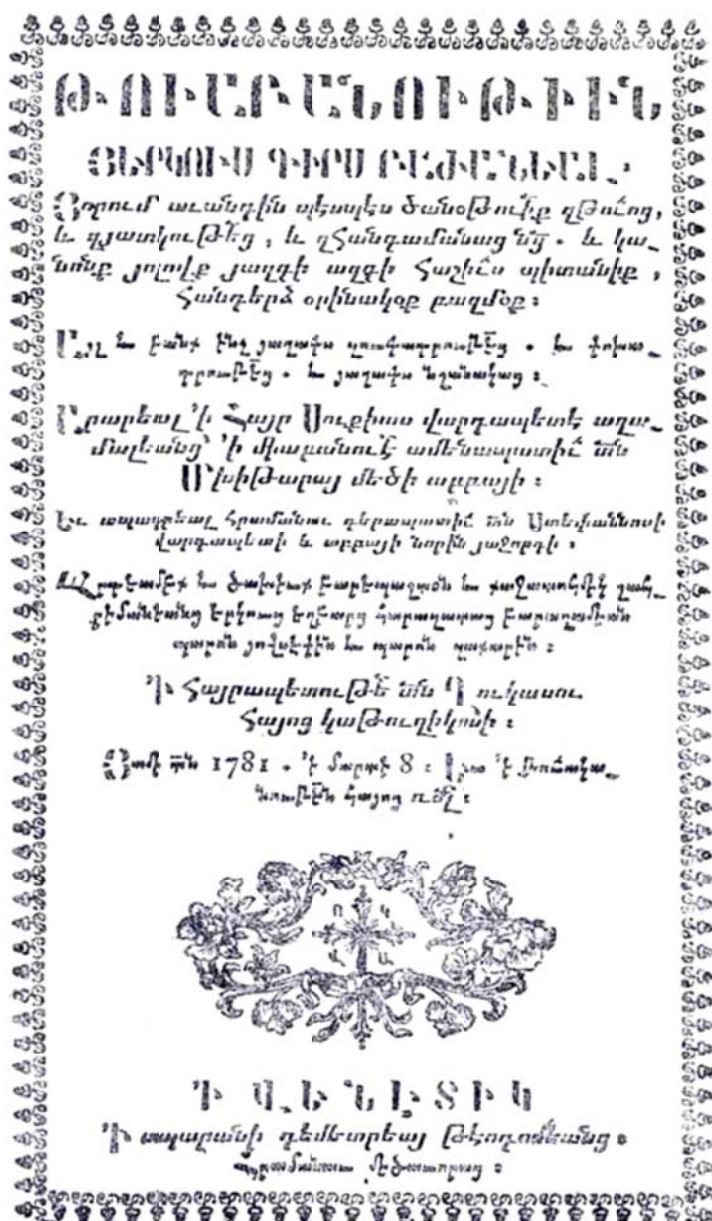
"Our people are not familiar well with calculations, so they do their counting with their fingers, and some others do it with peas, some with sand and so on....

We decided to present in this arithmetic not only the rules and principles in a manner that it is clearly and simply expressed, but also we added more exercises and examples and proofs if necessary to prove them"⁴⁶.

Aghamaliants had good knowledge of addition, subtraction, multiplication and division signs but he did not use them.

Aghamaliants not only provides rules and principles and methods of calculations needed for everyday life in those days, especially for businessmen, but also provides various other rules that were innovations of the time.

⁴⁶ - S. Aghamaliants, Arithmetic, Venice, 1781, p.4.



Title page of “Arithmetic” by Aghamaliant

Aghamaliants's arithmetic consists of two volumes, the first volume contains three chapters in 292 pages and an appendix. The first part is allocated to counting system of four main operations.

When talking about the arithmetic, and following example is presented and how to read it :

	3		2		1		0
111	111	111	111	111	111	111	111

Which reads as:

One hundred and eleven thousand, one hundred and eleven million, million, million
 One hundred and eleven thousand, one hundred and eleven million, million
 One hundred and eleven thousand, one hundred and eleven million

Instead of symbol of zero (0) the word "zero" is used. It is noteworthy that at that time people in other territories wrote and read in the same way. They presented the mathematical definitions in the same time.

When talking about four major operations, all existing and proposed possible ways were presented. A new approach from the left is introduced for addition operation. Apart from division methods, the method of "Galera" is also examined. Reverse operational methods were used for testing subtraction and division operations. The second part is dedicated to fractions and decimals. The third part is for the analogy, partnership, composition, etc. The appendix consists of 48 pages devoted to different problems.

The second book is in 209 pages and the author has divided it into four parts. At the beginning of the first section there is a summary of the numbers, even, odd, square, cubical and whole numbers (ie numbers that are equal to the sum of its product like 6 as $1+2+3=6$, or 28 as in

$1+2+4+7+14=28$) and methods to find the greatest common divisor, find the root of the second and third integer numbers are provided.

The second part is allocated to arithmetic and geometric progressions, and the third section is devoted to whole numbers, the polygon, etc.

The fourth section is devoted to the logarithms.

10^0	10^1	10^2	10^3	10^4	10^5	10^6	10^7
1	10	100	1000	10000	100000	1000000	10000000
2	20	400	8000	160000	3200000	64000000	1280000000
3	30	900	27000	810000	24300000	729000000	21870000000
4	40	1600	64000	2560000	102400000	4096000000	167772160000
5	50	2500	125000	6250000	312500000	15625000000	781250000000
6	60	3600	216000	12960000	777600000	46656000000	2824295364000
7	70	4900	343000	24010000	1680700000	117649000000	8235430000000
8	80	6400	512000	40960000	3276800000	268435456000	21473836480000
9	90	8100	729000	67290000	5904900000	531441000000	47829690000000
10	100	10000	1000000	100000000	10000000000	1000000000000	100000000000000
11	110	12100	1771000	238321000	26136761000	2851886321000	313276877441000
12	120	14400	2196000	324864000	39813120000	4776326400000	57646075200000
13	130	16900	2704900	431010000	56121361000	7297386121000	94922369241000
14	140	19600	3349600	562406400	78344320000	10941893760000	151377799040000
15	150	22500	4095000	731250000	107812500000	16171875000000	242550937500000
16	160	25600	4913000	937760000	149900800000	23986304000000	383983616000000
17	170	28900	5832100	1176490000	199070410000	33748032100000	571786120810000
18	180	32400	6859600	1454184000	262144640000	47185928100000	846738273610000
19	190	36100	8052100	17821361000	3379443200000	64420524000000	1216708968100000
20	200	40000	9377600	2209984000	4300640000000	840900000000000	1681792000000000

The first page of the logarithms, arithmetic book of Aghamalian

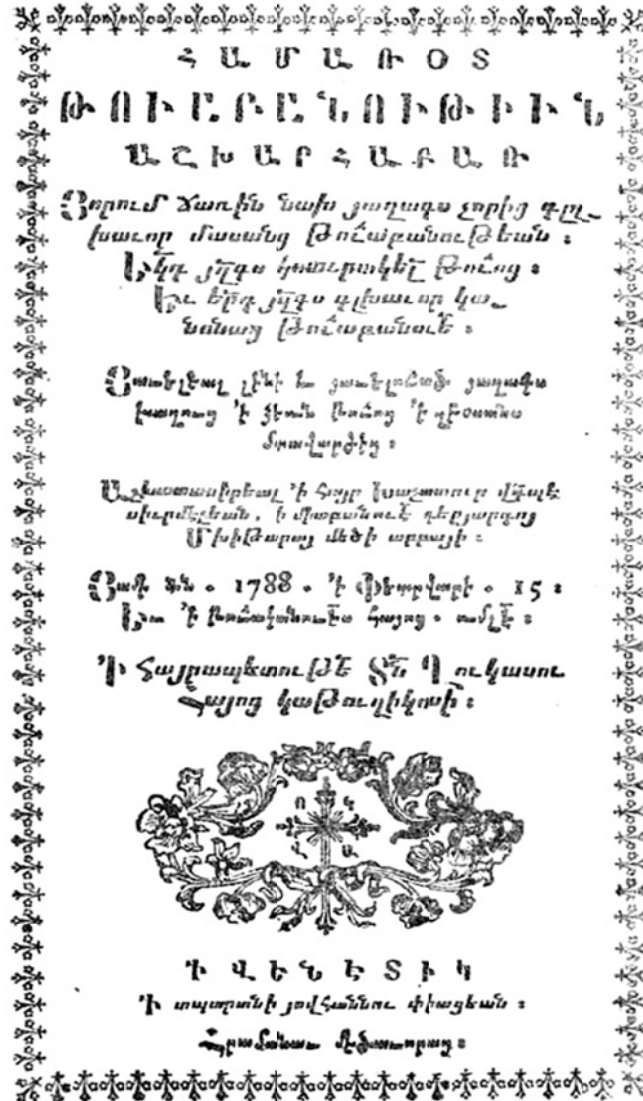
5-The fifth book:

The fifth book is "The application of computational techniques" by Minas Vartapet which was published in 1787 in Tercit.

6- The sixth book:

The sixth book is "Summary of arithmetic" by Khachatur Vartapet Surmelian, published in 1788 in Venice, which consists of 160 pages and three chapters. The content of this book is the same as the book of

Aghamaliants. Unlike Aghamaliants, Surmelian has used also Arabic and Turkish words.



Title page of Surmelian's book

7- The Seventh book:

"Geometry" by Sahak Prunian, is the seventh book printed in Venice in 1794 with 423 pages.

At the end of the book there are eight tables and 275 geometrical shapes and the letters of the Armenian alphabet are used to read them. Pre-page table of contents consists of a list of mathematical terms for some of them there are Arabic, Latin and Turkish equivalents.

The book includes a foreword in 4 pages and 24 pages of introduction at the beginning which the author raises some important issues there.

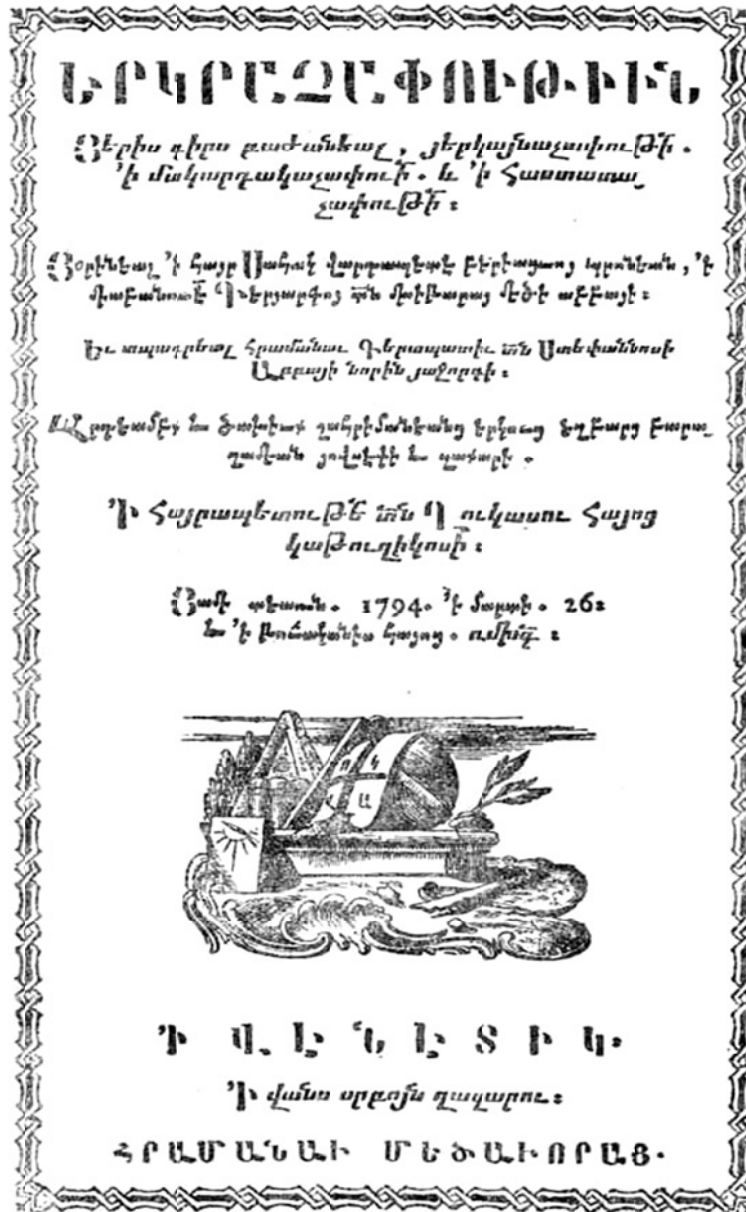
By emphasizing the importance of the theoretical and practical geometry, he expresses his long time interest in writing the geometry in the Armenian language, but for various reasons this had been disrupted.

The main purpose of this book is to present geometric content based on existing resources, in a language understandable even for beginners and self-learners. The first part of the book deals with geometry definitions.

Prunian's geometry is divided into different levels: theoretical and practical, basic and advanced, therefore it gives a detailed description. Let us see what is the purpose of Prunian's advanced geometry.

"The higher or advanced geometry deals with curved lines, ie lines that are not straight and not at all arc, but by bending continues as conic sections such as oval. It also speaks about volumes and areas which are created by the same lines. The first founder of the Advanced geometry is Apollonius, who was active in 244 BC, and Kardesius improved it.⁴⁷"

⁴⁷ - Sahak Prunian, Geometry, Venice, 1794, p. 6.



The title page of the book "Geometry" by Sahak Prunian

The first part of the second book is about squares, Part II is about polygons, Part III compares the areas, Part IV speaks about sections of areas and the appendix in 23 pages dedicated to solving problems.

The first part of the third chapter speaks about the definition of geometric volume of sphere, cone, cylinder, multi-dimensional geometric volumes and their similarities Part II, Part III to measure areas, the fourth to compare areas and appendix is devoted to solving problems.

8- The eighth book

"Trigonometry" by Sahak Prunian is the eighth Armenian printed book in mathematics that was published after his death in 1810 in Venice and includes two volumes and 104 pages. At the end of the book there are logarithmic tables for numbers 1 to 10,000 in 62 pages.

After the tables there are 28 theorems and the trigonometric problems and then two comparisons provided by Compass.

At the end, the trigonometric lines are presented on one page.

The book contains a two-page introduction in which the important role of trigonometry in the mathematical sciences is described.

The first book is dedicated to the straight lines in trigonometry and the trigonometric lines are defined, the following terms are used:

(Dsots) = sin

(Hamadsots) = cos

(Shoshapogh) = tg

(Shoshapoghakits) = cotg

(Hatanogh) = sec

(Hatoghakits) = cosec

For the first time Prunian uses all mathematical signs and symbols (addition, subtraction, multiplication, division, etc.) in the Armenian printed mathematical literature.

The second part is allocated to logarithm of natural numbers, the third part is about the triangle solving problems related to rectangular and non-rectangular triangles.

The second volume is dedicated to digital trigonometry. In the first part is about the definitions, relation between sides and angles. The second part is devoted to solving the triangle. In the appendix analog compass and its use is discussed.

Section 12

Armenian mathematics in the 19th and 20th centuries

As mentioned in previous chapters, from the 14th century, the Armenian scientific centers were located in near and far countries. In mainland Armenia due to economic and social non-stability, establishing scientific research centers was not possible. Armenians living in different countries who had comparatively better living conditions, paid attention to this matter. Various mathematical books were written or translated. In the 19th century 84 textbooks were published, that most of the authors were Armenian mathematicians. Especially "Geometry" and "Trigonometry" by Sahak Prunyan, "Honest mathematics" by Ghucas Terteriants and others were important.



Yerevan State University

Mathematics among Armenians once again had glorious days in the twentieth century, especially after the 1920, and this has been associated with the name of the Yerevan State University.



Academy of Sciences of Armenia

Teaching mathematics begins at the School of Science and Engineering of Yerevan State University from 1921 and training of professionals in mathematics at physics-mathematical department of pedagogical school since 1924. In 1930, Yerevan State University was divided into several independent Institutes. Faculty of Engineering was transformed into the Polytechnic Institute. Faculty of Pedagogics was transformed to Teacher Training Institute, one of the four schools was named the Faculty of Technical Physics, which trains teachers in mathematics and physics.

In 1933, Yerevan University had 5 faculties and 15 departments, and The School of Mathematics-physics was one of them, which in 1959 was divided into two schools of mathematics- mechanics and physics. The first one was in charge of training specialists in mathematics and mechanics,

but the second one trained physicist and astronomers. School of mathematics and physics passed four periods based on the structure and goals of the faculty and staff and student class:

First period: from 1924 to 1933. During this period, the school had educational nature and prepared teachers in mathematics and physics. The first professors of mathematics in this period were: Alexander Hacopian, Arshak Tonian, Hovhannes Navakatikian, Bahadur Bhadrian, Zbet Khojantian, Ashot Megerdichian and others.

Second period: from 1933 to 1941-42. The faculty got more collegiate and university nature with a five-year period and began to train math teachers not only for high schools but also experts needed to teach in higher education institutions. The majority of faculty members were the first college graduates and were graduates of the doctoral programs from the universities of Moscow and Leningrad. In these years, the following persons have passed doctoral programs of the universities: G. Petrosian (theoretical physics and the history of mathematics), N. Petrosian (equations of mathematical physics), V. Saghatelian (theory of functions), A. Shahinian (Theory of functions), N. Gasparian (geometry), S. Abajian (algebra), G. Toumanian (differential equations) and so on. Among the young professors of this period we should mention Kh. Torgomian who graduated in 1929 from the faculty and was working as assistant professor since 1927 and then became professor. After a short time he became one of the most eminent professors in the field of mathematical physics and mathematical analysis and brilliantly directed higher mathematics in the Polytechnic Institute for 30 years. In 1938, the outstanding young mathematician D. Mashurian from Moscow University was invited to teach in the field of differential and integral equations in Yerevan. He got killed in battle during the Second World War.

In the early 1940s, professors of mathematics in Armenia, in general belonged to the generation of young mathematicians. They passed the first higher education in doctoral level and used their experience in the fields of mathematics and scientific researches.

In order to promote scientific research level some changes happened in the structure of the chairs in the faculties. Until 1938 there were two permanent chairs in the faculty, Chair of Mathematics (headed by Arshak Tonian) and Chair of Physics (headed by Hovhannes Navakatikian). In January 1938 the faculty had seven chairs, three of them on mathematics (differential equations headed by N. Petrosian, geometry headed by A. Shahinian, higher algebra headed by Z. Khojantian).

Third period: This period is 1943-59. In this long period, the economic development and science begins. Technology had great achievements, in particular the Academy of Sciences of Soviet Armenia was established in 1943 and by development of science and research networks the need for training experts in mathematics and researchers was under discussion. In the field of education and training specialists and professors of mathematics, significant activities were done. Significant reforms were carried out in the School of Mathematics, seminars and lectures were developed and implemented. Various professional fields were established. Thus, in the periods, general mathematicians were trained in the school but in this period, they were graduated in various specialties, such as the theory of functions, differential equations, differential geometry, probability theory, mathematical statistics, mathematics-computers and so on. By specialization in the field of mathematical areas, separate chairs were established for each field, including the chair of the history of physics and mathematics headed by G. Petrosian. In the years 1944-60 post-doctoral scientific degrees were awarded to a number of mathematicians, including A. Shahinian, M. Jerbashian, S. Mergelian, H. Badalian, and G. Petroian.

At the faculty of Mathematics of Yerevan University the first students graduated in 1929, then in 1930, and until 1959 the faculty had total of 194 graduates. During 1929-1959 a total of 1,000 people graduated in the fields of mathematics, physics, mechanics and astronomy.

The fourth period: It starts from 1959. In general, the Armenian mathematicians have been researching in the field of theory of functions. The prominent figures in mathematics during this period are A. Shahinian, M. Jerbashian. S. Mergelian. By 1960, the first one published a total of 30, the second 60 and third 35 scientific works.

Thanks to the efforts of Yerevan State University and the Academy of Sciences of Armenia, Yerevan became one of the pioneers of the mathematical sciences in the field of theory of functions and complex numerical analysis in former Soviet republics. In 1965, the International Congress of Mathematics was held in Yerevan. Since 1965 the journal of "Matematica" (mathematics) is being published:



A sample issue of Matematica- ISSN: 00002-3043 and 1829-1163
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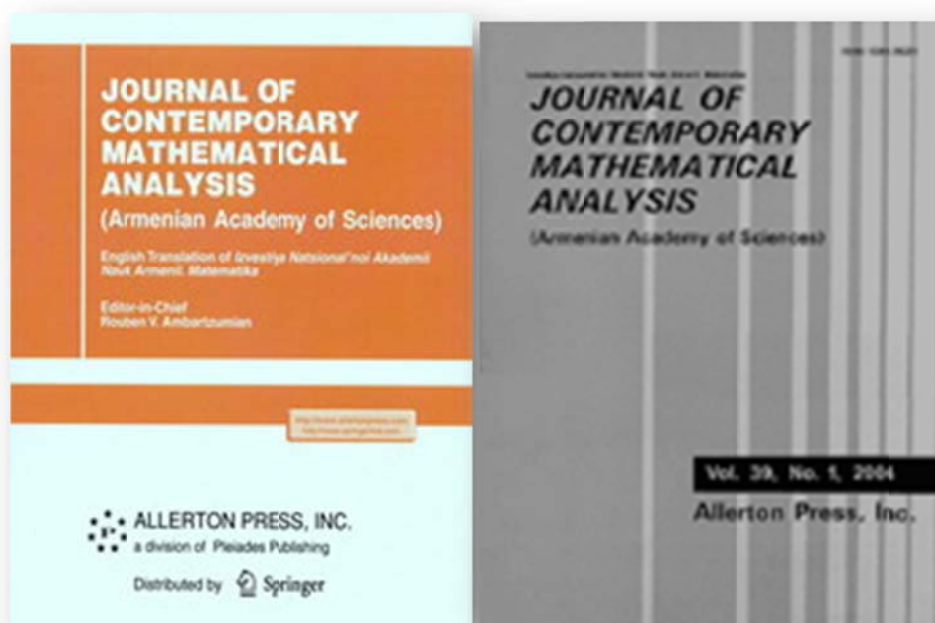
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Samples issue of Journal

In 2008 online “Armenian Journal of Mathematics” (Semiyearly Scientific Peer Reviewed all-electronic Journal) was established by the National Academy of Sciences of Armenia. It is published semiyearly, 10 issues are published so far (Volume 7, Issue 1, 2015).

<http://www.flib.sci.am/eng/journal/Math/>

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Section 13

Prominent Armenian mathematicians in Armenia and Diaspora

In the following text a brief biography of several Armenian mathematicians is presented:

1. **Alexander (Sumbat) Abian** (January 1, 1923—July 1999) was an Iranian-born Armenian-American mathematician who taught for over 25 years at Iowa State University and became notable for his frequent posts to various Usenet newsgroups. Abian was born in Tabriz, Iran, and was of Armenian ethnicity. After earning an undergraduate degree in Iran, he emigrated to the United States in 1950, where he received a master's degree from the University of Chicago. Abian then obtained a Ph.D. from the University of Cincinnati, where he wrote a dissertation on a topic in invariant theory under the direction of Isaac Barnett. Before teaching posts in Tennessee, New York, Pennsylvania, and Ohio, he joined the faculty of Iowa State in 1967. He wrote three books and published more than two hundred papers. He retired in 1993. Abian gained a degree of international notoriety for his claim that blowing up the Moon would solve virtually every problem of human existence. He made this claim in 1991 in a campus newspaper. Stating that a Moonless Earth wouldn't wobble, eliminating both the seasons and its associated events like heat waves, snowstorms and hurricanes. Refutations were given toward that idea by NASA saying that part of the exploded Moon would come back as a meteorite impacting the Earth and causing sufficient damage to extinguish all life, while restoring the seasons in the process. Abian said that "Those critics who say 'Dismiss Abian's ideas' are very close to those who dismissed Galileo." This claim and others, made in thousands of Usenet posts during the last portion of his life, gained Abian mention (not

entirely favorable) and even interviews in such diverse publications as Omni, People, and The Wall Street Journal.

2. **Dr. Megerdich Toomanian:** Dr. Toomanian was born in 1943 in the village of Chigan Frieden (Isfahan) Iran. He got is Ph.D. In mathematics from the University of Southampton in 1975. Professor of the Faculty of Mathematical Science of Tabriz University, Prof., Corr.-member of Iranian Academy of Sciences, Faculty of Mathematical Science, Tabriz University, Tabriz, Iran Foreign member of the National Academy of Sciences of Armenia since 2008. He has 10 scientific works so far.

3. **Dr. Vazgen Avanesian (1927 Qazvin, Iran- 2007 Strasburg, France),** graduated from the Sorbonne in Paris, for a long time, Professor and Head, Department of Mathematics, University of Strasbourg, France,. Dr. Avanesian for a while was teaching at the National University of Iran and published the first book of new mathematics in Persian language, which was the only textbook for long time. Unfortunately, due to a dispute with the management of time, again he returned to France and the University of Strasbourg. Dr. Avanesian had published numerous articles in algebra, linear algebra in, international mathematical journals.

4. **Dr. Vrezh Amirkhanian,** graduate of algebra from England. He was professor of mathematics at the University of Ahvaz (Iran) before the Islamic revolution. After the revolution he taught at the University of Qazvin (Iran). He has published several articles in international journals in the field of algebra. He is also a member of the association of mathematics, Iran.

5. **Garegin Petrosyan,** Mathematician; Physical and Mathematical Sciences, Professor, Soviet scientist. He was born in 1902, September 10,

Goris, Armenia. 1929. He graduated from the Faculty of Physics and Mathematics, Yerevan State University in 1929 and Postgraduate studies from Moscow Lomonosov State University of Moscow in 1934.

1934-1936. Dean of the Faculty of Physics and Mathematics at the Faculty of Educational Affairs. He was awarded the rank of assistant professor in 1938. He was president of Yerevan State University in 1938-1941, while he was in charge of the Theoretical Physics Department.

During World War II, in 1941 G. Petrosyan voluntarily went to the front, got wounded, returned home and joined the newly established Academy of Sciences of Armenia, Yerevan and worked as a secretary in the office of the president up to 1949

1949-1957 - Petrosyan took a responsibility in the State University as president. He founded the University's Faculty of International Relations, Department of History of Physics and Mathematics, which he headed until 1956. He defended his postdoctoral dissertation in 1955. In 1964 He was awarded the title of professor. In 1965 he was elected as correspondent member of the International Academy of History of Science and Technology.

Petrosyan was engaged in the history of sciences since early 1940. His works are mostly related to mathematics, the mathematics of the ancient and medieval history.

He has studied and evaluated the works of the ancient and medieval Armenian mathematicians. Some of his books are: "Mathematics of ancient and Middle Ages" (Yerevan, 1959), "History of mathematics in medieval Armenia (articles and publications)" (Yerevan, 1986). G. Petrosyan is also co-author of "Anania Shirakatsi bibliography" (Yerevan, 1979) He was also originator and Editor-in-Chief of the "History of Natural sciences and technology in Armenia" series of books.

1938-1947 and 1951-1955 he was elected member of the Supreme Soviet (Parliament) of Soviet Armenia, and in 1954-1958 member of USSR Supreme Soviet.

1967. G.Petrosyan was awarded the USSR Honored Scientist title. He was awarded the Order of the Red Banner of Labor and many other medals.

He died in December 12,1997 in Yerevan.

6. **Rouben V. Ambartzumyan** (born 1941) is an Armenian mathematician and Academician of National Academy of Sciences of Republic of Armenia. He worked in Stochastic Geometry and Integral Geometry where he created a new branch, combinatorial integral geometry. The subject of combinatorial integral geometry received support from mathematicians K. Krickeberg and D. G. Kendall at the 1976 Sevan Symposium (Armenia) which was sponsored by Royal Society of London and The London Mathematical Society. In the framework of the later theory he solved a number of classical problems in particular the solution to the Buffon Sylvester problem as well as the Hilbert's fourth problem in 1976. He is a holder of the Rollo Davidson Prize of Cambridge University of 1982. Rouben's interest in Integral Geometry was inherited from his father. Nobel prize winner Allan McLeod Cormack Laureate for Tomography wrote: "Ambartsumian gave the first numerical inversion of the Radon transform and it gives the lie to the often made statement that computed tomography would have been impossible without computers". Victor Hambardzumian, in his book "A Life in Astrophysics", wrote about the work of Rouben V. Ambartzumian, "More recently, it came to my knowledge that the invariance principle or invariant embedding was applied in a purely mathematical field of integral geometry where it gave birth to a novel, combinatorial branch.

7. **Mkhitar Jerbashyan**; (11 September 1918 – 6 May 1994) was a notable Armenian mathematician, who made significant contributions to the constructive theory of functions, harmonic analysis, theory of analytic functions and a fundamental contribution to the classical theory of

univalent analytic functions. He was born in Yerevan in a family of refugees from the town Van of Western Armenia escaping from the Armenian Genocide of 1915 in Turkey. Mkhitar Djrbashian created some well-known mathematical theories (see, e.g.) and did everything possible for the development of Armenian Mathematical School to the high international standards in many branches of mathematics.

8. **Leonid Khachiyan** (May 3, 1952 – April 29, 2005) was a Soviet mathematician of Armenian descent who taught Computer Sciences at Rutgers University. He was most famous for his Ellipsoid algorithm (1979) for linear programming, which was the first such algorithm known to have a polynomial running time. Even though this algorithm was shown to be impractical due to the high degree of the polynomial in its running time, it has inspired other randomized algorithms for convex programming and is considered a significant theoretical breakthrough. Khachiyan was born in St. Petersburg and moved to Moscow with his parents at the age of 9. There he later earned a Ph.D. in computational mathematics in 1978 and a D.Sc. in computer science in 1984, both from the Computing Center of the USSR Academy of Sciences. In 1982 he won the prestigious Fulkerson Prize from the Mathematical Programming Society and the American Mathematical Society for outstanding papers in the area of discrete mathematics. Prior to moving to the United States in 1989, Khachiyan held a series of research and teaching positions at the Computing Center of the USSR Academy of Sciences and the Moscow Institute of Physics and Technology. In 1989 he joined Cornell University's School of Operations Research and Industrial Engineering as a visiting professor and had been at Rutgers since 1990. After moving to the United States, Khachiyan's work continued some of its old ideas, as he worked on the complexity of maximal volume inscribed ellipsoids and wrote a paper on rounding polytopes, adding some

new ones. He wrote a series of papers with Bahman Kalantari on various matrix scaling and balancing problems.

9. **Yervand Kogbetliantz** (born February 22, 1888 in Armenia – died of cancer in 1974 in Paris, France) Dr. Kogbetliantz was an Armenian/American mathematician and the first president of the Yerevan State University. He left Russia in 1918. He received a Doctorate in mathematics from the University of Paris in 1923. His mathematical work was mainly on infinite series, on the theory of orthogonal polynomials, on an algorithm for singular value decomposition which bears his name, on algorithms for the evaluation of elementary functions in computers, and on the enumeration of prime elements of the Gaussian integers. He also invented a three-dimensional version of chess, and was working with Bobby Fischer on a game of chess for three people. When he first came to America (1941) he taught Mathematics at Lehigh University. In the early 1950's he was a consultant for IBM in New York City and taught at Columbia University. Prior to moving back to Paris and retiring he was a professor at Rockefeller University.

10. **Sergey Mergelyan** (born 19 May 1928 in Simferopol, Crimea, Soviet Union; died 20 August 2008 in Los Angeles, USA; buried at the Novodevichie Memorial Cemetery in Moscow, Russia) was a Soviet Armenian scientist, an outstanding mathematician, who is the author of major contributions in Approximation Theory. The modern Complex Approximation Theory is based on Mergelyan's classical work. He graduated from Yerevan State University in 1947. When he was just 20, in Moscow Steklov Mathematical Institute awarded him the USSR Doctor of Science degree in mathematics in addition to his Ph.D. Up today this is an absolute record of getting the highest scientific degree (Doctor of Science) at such young age in former USSR and present Russia. In 1952 he was awarded USSR

State Prize. When he was just 24 he became corresponding member of the Academy of Sciences of the USSR (now Russian Academy of Sciences), which, from the point of view of young age, is yet another absolute record among USSR scientists. He has been a symbol of a young scientist in former USSR. Indira Gandhi, among other famous people in USSR and abroad, has been a friend of Mergelyan from early 1950s. In 1978, after her official visit to Moscow, Gandhi had also a private visit to Yerevan just as a guest of Mergelyan.

Mergelyan played a leading role in establishing Yerevan Scientific Research Institute of Mathematical Machines -commonly known as Mergelyan Institute of Mathematical Machines- in 1956. He became the first director of the institute.

His works include theory of functions of complex variables, theory of approximation, and theory of potential and harmonic functions. In 1951 he formulated and proved the famous result from complex analysis called Mergelyan's theorem. This theorem of Mergelyan solved an old classical problem. Several years later he solved another famous problem, the Sergei Natanovich Bernstein Approximation Problem. Mergelyan also has many important results in other areas of complex analysis including the theory of pointwise approximations by polynomials.

11. **Mamikon A. Mnatsakanyan** is Project Associate at Project Mathematics at California Institute of Technology. He received a Ph.D. in physics in 1969 from Yerevan State University, where he became professor of astrophysics. As an undergraduate he specialized in the development of geometric methods for solving calculus problems by a visual approach that makes no use of formulas, which he later developed into his system of **visual calculus**. In 2010 he was nominated by Caltech for the Ambartsumians International Prize, awarded annually by the President of Armenia, for his contributions in the field of theoretical

astrophysics. In 1959 he discovered a new proof of the Pythagorean theorem.

12. Ashot Petrosyan (born June 2, 1930 in Vardenis region, Armenia; died February 23, 1998 in Dilijan, Armenia) was a Soviet Armenian mathematician. He completed his PhD in Computational Mathematics in 1964 under the supervision of Julius Anatolyevich Schrader. He was a founding member of the Mergelyan Institute of Mathematical Machines and the Computing Center of the Armenian National Academy of Sciences. He also contributed to the development of several generations of advanced digital computer systems in Armenia, including the Nairi (computer) and ES EVM. Ashot V. Petrosian was born in 1930 in a small village near Vardenis, Armenia. He finished high school as a valedictorian in 1949 in Dilijan, Armenia, where his parents settled after escaping the massacre of Armenians in eastern Turkey back in 1915. He studied mathematics and graduated with honors from the Yerevan State University, faculty of Physics and Math in 1954. He taught Math courses at the same University until 1955, when he was admitted to Moscow State University to pursue a Ph.D. degree (under supervision of Lazar Lyusternik). Upon completion of his studies he was offered a position at the Steklov Institute of Mathematics, however he decided to return to his native country of Armenia. In 1957 he was appointed to serve as a Chief Engineer and then as Director of Mathematical Division at the Yerevan Computer Research and Development Institute (YCRDI, known as Mergelyan Institute). During his tenure at the YCRDI, the institute became one of the largest producers of computer equipment in the former USSR. He also worked as the Vice-Principal (1963–65) and Principal (1965–70) of the Institute for Informatics and Automation Problems (IIAP), formerly known as the Computing Center of the Armenian National Academy of Sciences. During his scientific career, Prof. Petrosian taught various

mathematics courses at the Yerevan State University (1957–78) and at the Yerevan Polytechnic Institute (1978–86). He has authored several textbooks, patents, and monographs in the areas of computational mathematics, algorithmic information theory, automata and discrete mathematics. He has edited five volumes of the Proceedings of the Computing Center of the Armenian National Academy of Sciences and served as a Ph.D. adviser to over 20 graduate students, mainly in the Graph Theory field.

13. **Rom Varshamov** (Born April 9, 1927 in Tbilisi; Died August 24, 1999 in Moscow) was a Soviet Armenian mathematician who worked in Coding theory, especially on error-correcting codes and Number theory. Varshamov studied in Tbilisi with Arnold Walfisz (where he was Georgian students' champion in the 100 meters), as well as in Tomsk. After that, he was a researcher in Moscow at the Steklov Institute of Mathematics with Ivan Matveyevich Vinogradov, especially on Number theory and Coding theory, and the Ministry of Radio Engineering (working in Cryptography). In 1957 he proved the Gilbert-Varshamov bound for linear codes (independently of Edgar Gilbert who proved the non-linear part). From 1968 he worked in Yerevan and was director of the Computer Center (now Institute for Informatics and Automation Problems) of the Academy of Sciences of the Armenian SSR. He was author and co-author of many scientific articles and also a member of the Armenian National Academy of Sciences.

14. **George Piranian** (May 2, 1914 – August 31, 2009), was a Swiss-American mathematician of Swiss and Armenian descent. Piranian was internationally known for his research in complex analysis, his association with Paul Erdős, and his editing of the *Michigan Mathematical Journal*.

15. **Dmitry Mirimanoff** (13 September 1861, Pereslavl-Zalessky, Russia – 5 January 1945, Geneva, Switzerland). His parents were Semion Mirimanovitch Mirimanoff and Maria Dmitrievna Rudakova. He was a great grandson of David A. Mirimanian (later Mirimanoff), a member of an old Armenian family settled in Georgia and an honorary citizen of Tiflis (now Tbilisi) became a doctor of mathematical sciences in 1900, in Geneva, and taught at the universities of Geneva and Lausanne. Mirimanoff made notable contributions to axiomatic set theory and to number theory (relating specifically to Fermat's last theorem, on which he corresponded with Albert Einstein before the First World War). In 1917, he introduced, though not as explicitly as John von Neumann later, the cumulative hierarchy of sets and the notion of von Neumann ordinals; although he introduced a notion of regular (and well-founded set) he did not consider regularity as an axiom, but also explored what is now called non-well-founded set theory and had an emergent idea of what is now called bisimulation.

16. **Guy Terjanian** is a French mathematician who has worked on algebraic number theory. He achieved his Ph.D under Claude Chevalley in 1966, and at that time published a counterexample to the original form of a conjecture of Emil Artin, which suitably modified had just been proved as the Ax-Kochen theorem. He is ethnically Armenian. In 1977, he proved that if p is an odd prime number, and the natural numbers x , y and z satisfy then $2p$ must divide x or y .

**A brief list of Armenian mathematicians, members
of Institute of Mathematics, National Academy of
Sciences of the Republic of Armenia**

17. Sergey Afyan, Associate Prof., Faculty of Physics, Yerevan State University.

18. Sargis Aleksanyan, Ph.D, Junior Scientific Researcher of Institute of Mathematics of NAS.

19. Silva Andriyan, Lecturer, Department of Higher Mathematics, Armenian Agricultural Academy.

20. Levon Arabajyan, Prof. Dr., Armenian State Pedagogical University.

21. Norair Arakelyan, Academician of the NAS of Armenia, Academician-Secretary of the Division of Mathematical and Technical Sciences, Member of the Presidium of the NAS RA.

22. Avetik Arakelyan, Researcher at Institute of Mathematics, NAS of Armenia.

23. Rafik Aramyan, Institute of Mathematics Armenian National Academy of Sciences; Russian-Armenian (Slavonic) University.

24. Gurgen Asatryan, Armenian State Pedagogical University.

25. Karen Avetisyan, Associate Prof., Faculty of Physics Yerevan State University.

26. Armenak Babayan, Researcher, Faculty of Mathematics Armenian State Engineering University.
27. Artur Barkhudaryan, Institute of Mathematics of NAS RA, Yerevan State University.
28. Rafayel Barkhudaryan, Director of the Institute of Mathematics.
29. Grigor Barsegyan, Institute of mathematics of the National Academy of Sciences of Armenia.
30. Anahit Chubaryan, Associate Prof., Faculty of Informatics and Applied Mathematics Yerevan State University.
31. Vahram Dumanyan, Associate Professor, Department of Mathematical Methods and Modeling, Yerevan State University.
32. Sergo Episkoposyan, Lab. Assistant in State Engineering University of Armenia.
33. Anahit Galstyan, Ph.D., Institute of Mathematics Armenian National Academy of Sciences.
34. Ashot Galumyan, Associate Prof., Faculty of Physics, Yerevan State University.
35. Levon Gevorgyan, Ph.D, Department of Mathematics, State Engineering University of Armenia.
36. Gegham Gevorkyan, Corr. member of Armenian Academy of Sciences.

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37. Tatevik Gharibyan, Ph. D, Scientific Researcher of Institute of Mathematics of NAS RA.
38. Gayane Ghazaryan, Assistant Professor, Faculty of Physics, Yerevan State University.
39. Hayk Ghazaryan, Professor, Faculty of Economics, Yerevan State University.
40. Mamikon Ginovyan, Professor Dr., Institute of Mathematics Armenian National Academy of Sciences.
41. Smbat Gogyan, Researcher, Department of Real Analysis, Armenian Academy of Sciences.
42. Martin Grigorian, Prof. Dr., Faculty of Physics, Yerevan State University.
43. Hrachik Hairapetyan, Professor Dr., Department of Mathematics, State Engineering University of Armenia.
44. Aram Hakobyan, Asistant Professor, Faculty of Mathematics, Yerevan State University.
45. Gagik Hakobyan, Associate Prof., Faculty of Physics, Yerevan State University.
46. Gurgen Hakobyan, Associate Prof., Faculty of Mathematics, Yerevan State University.

47. Hakop Hakopyan, Assoc. Professor Dr., Faculty of Informatics and Applied Mathematics, Yerevan State University.

48. Yuri Hakopyan, Professor Dr., Faculty of Informatics and Applied Mathematics, Yerevan State University.

49. Anahit Harutyunyan, Associate Prof., Faculty of Informatics and Applied Mathematics, Yerevan State University.

50. Gohar Harutyunyan, Ass. Professor, Faculty of Informatics and Applied Mathematics, Yerevan State University.

51. Tigran Harutyunyan, Associate Prof., Department of Differential Equations, Faculty of Mathematics, Yerevan State University.

52. Artur Hovhannisyan, Prof. Dr., Faculty of Radiophysics, Yerevan State University.

53. Gro Hovhannisyan, Professor, Institute of Mathematics of Armenian National Academy of Sciences.

54. Nune Hovhannisyan, Scientific Researcher, Institute of Mathematics Armenian National Academy of Sciences.

55. Armen Jerbashyan, Institute of Mathematics National Academy of Sciences of Armenia.

56. Armen Kamalyan, Assistant Prof., Department of Differential Equations, Faculty of Mathematics, Yerevan State University.

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57. Grigori Karagulyan, Prof. Dr., Institute of Mathematics of Armenian National Academy of Sciences.
58. Arman Karapetyan, Institute of Mathematics Armenian National Academy of Sciences.
59. Aghavard Khachatryan, Professor, Head of Department of Higher Mathematics, Armenian Agrarian State University.
60. Aram Khachatryan, Associate Prof., Faculty of Radiophysics, Yerevan State University.
61. Khachatur Khachatryan, Ph.D., Scientific Researcher of NAS.
62. Anatoly Kitbalyan, Associate Prof., Faculty of Physics, Yerevan State University.
63. Mher Martirosyan, Associate Professor, Department of Math. Analysis, Faculty of Mathematics, Yerevan State University.
64. Lavrenty Matevosyan, Associate Prof., Department of Algebra and Geometry, Faculty of Mathematics, Yerevan State University.
65. Naira Mesropyan, Associate Prof., Faculty of Mathematics, Yerevan State University.
66. Gagik Mikaelyan, Associate Prof., Faculty of Mathematics Yerevan State University.
67. Hayk Mikayelyan, Researcher.

68. Levon Mikayelyan, Associate Prof., Department of Applied Analysis, Faculty of Informatics and Applied Mathematics, Yerevan State University.

69. Yervand Mkrtchyan, Associate Prof., Faculty of Informatics and Applied Mathematics Yerevan State University.

70. Maksim Muradyan, Prof. Dr., Yerevan State Institute of Economy.

71. Sahak Narimanyan, Associate Prof., Faculty of Mathematics, Yerevan State University.

72. Karen Navasardyan, Assistant Prof., Faculty of Informatics and Applied Mathematics, Yerevan State University.

73. Varazdat Navoyan, Associate Prof., Faculty of Radiophysics, Yerevan State University.

74. Ernst Nazaryan, Faculty of Mathematics Yerevan State University.

75. Anry Nersesyan, Professor Dr., Institute of Mathematics Armenian National Academy of Sciences.

76. Vagarshak Ohanyan, Associate Prof., Faculty of Radiophysics, Yerevan State University.

77. Victor Ohanyan, Associate Professor, Faculty of Mathematics, Yerevan State University.

78. Stella Papayan, Lecturer, Department of Mathematics Armenian State Engineering University.

79. Albert Petrosyan, Professor Dr., Faculty of Mathematics and Mechanics, Yerevan State University.

80. Araik Poghosyan, Associate Prof., Yerevan State Institute of Economics.

81. Arnak Poghosyan, Institute of Mathematics of Armenian National Academy of Sciences, Yerevan State University.

82. Michael Poghosyan, Associate Prof., Faculty of Mathematics and Mechanics, Yerevan State University.

83. Artur Sahakyan, Corresponding Member of Armenian National Academy of Sciences Dean of the Faculty of Mathematics and Mechanics of Yerevan State University .

84. Borik Sahakyan, Associate Prof., Faculty of Mathematics, Yerevan State University.

85. George Sahakyan, Artsakh State University Dean of the Faculty of Physics and Mathematics.

86. Vahan Sarafyan, Prof., Faculty of Mathematics, Yerevan State University.

87. Arpi Stepanyan, Scientific secretary of the Institute of Mathematics.

88. Alexandr Talalyan, Academician of the NAS of Armenia, Dr. Prof., Head of the Department of Real Analysis, Institute of Mathematics of National Academy of Sciences of Armenia.

89. Arakel Taslakyan, Associate Prof., Faculty of Mathematics, Yerevan State University.

90. Liparit Tepoyan, Dr., Department of Differential Equations, Faculty of Mathematics, Yerevan State University.

91. Vardan Tepoyan, Junior Researcher.

92. Ashot Vagharshakyan, Professor, Institute of Mathematics of Armenian National Academy of Sciences.

93. Karen Yagdjan, Dr., Institute of Mathematics Armenian National Academy of Sciences.

94. Bagrat Batikyan, Professor Dr., Institute of Mathematics Armenian National Academy of Sciences.

95. Sargis Hakobyan, Associate Prof., Faculty of Radiophysics, Yerevan State University.

96. Ishkhan Khachatryan, Professor, Yerevan State University.

97. Valeri Martirosyan, Professor Dr., Yerevan State University.

98. Vidok Musoyan, Prof. Dr., Department of Mathematical Analysis, Faculty of Mathematics, Yerevan State University.

99. Stella Safaryan, Lecturer, Faculty of Physics, Yerevan State University.

100. Romen Shahbaghyan, Professor, Faculty of Mathematics, Yerevan State University.

101. Nazaret Tovmasyan. Doctor of Sciences, 1967, Novosibirsk, Title "On the theory of boundary value problems for elliptic systems of the second order

Appendices

Appendix 1

Mathematical problems of Anania Shirakatsi

First problem: I heard from my father that when Armenians were fighting against Persians, great bravery was done by Zorak Kamsarakan. He attacked the army of Persians three times in a month. At the first time he slaughtered a half of the army. At the second time he slaughtered a quarter of the army. At the third time he slaughtered $1/11$ part of the army. All the rest (280 men) deserted to Nakhijevan. Now, with the number of the rest you must count the number of the Persian army.

Answer: There were 1760 riders before the slaughter.

Second problem: One of my relatives left for Bahl and bought pearls at very good price. When he arrived to Gandzak, he sold the half of the pearls with fifty Drams for every pearl. Then he went to Nakhijevan and sold the quarter of the pearls at seventy Drams for each pearl and in Dvin [he sold] at fifty Drams per pearl. When he came to Shirac, he had twenty-four pearls. Now figure out, how many pearls he had and what their price was.

Answer: He had 144 pearls and their price was 6720 Drams.

Third problem: I heard from [my] teacher that robbers entered the treasury of Markian and robbed a half of the treasure. Treasurers found only 421 Cendinars and 3600 Dahekans [1 Cendinar=7200 Dahekans]. Now, figure out, how much was all the treasure.

Answer: The treasure was 1686 Cendinars.

Fourth problem: The wages of the Saint Sofia friars was distributed so that the deacons got $1/5$, the priests got $1/10$, the bishops got 200 liters, all the rest got 2000 liters. Now figure out how many liters are the wages of Saint Sofia friars.

Answer: The wages of the friary is about 3200 liters.

Fifth problem: The wages of the officers were paid. One quarter is given to respectables, $1/8$ to seniors and 150 Cendinars is given to another riders. Now figure out! How much is the wage of the officers?

Answer: The wage of the officers is 240 Cendinars.

Sixth problem: There was a lettuce in my garden. A Roman entered my garden to take a walk. He ate $1/5$ and $1/15$ of the lettuce. I knew that he was greedy and I sent him away from my garden. I counted the remaining lettuces. There were 110 lettuces. Now figure out! How many lettuces were there and how much did the Roman eat.

Answer: There were 150 lettuces and the Roman ate 40 lettuces.

Seventh problem: I was in Marmet, in the lands of Kamsarakans. I went to the bank of the river Akhuryan. I saw shoal of fish. I threw a net and caught the half, the quarter and the $1/7$ of the fish. The 45 remaining of them escaped from the net. Now figure out! How many fish were in the shoal.

Answer: There was 420 fish in the shoal.

Eighth problem: When the Armenians were in revolt against Persians, Zorak Kamsarakan killed Suren. One of the noble men of Armenians sent an ambassador to the Persian king to tell him about the killing. He [The ambassador] was passing 50 miles in a day. After 15 days Zorak Kamsarakan was informed about it and sent his men to catch the ambassador. They were passing 80 miles in a day. Now figure out when they caught the ambassador.

Answer: They caught him 25 days after Zorak Kamsarakan was informed about treachery.

Ninth problem: Kamsarakans were on hunting in Gen. They hunted many animals. They gave me a wild boar. It was very big, so I scaled it. I figured out that its entrails were quarter of its weight, the head was $1/10$, haunches were $1/20$, its fangs were $1/90$ of its weight. The trunk was 212 liters. Now, figure out! How much was the wild boar weight?

Answer: The wild boar was 360 liters.

Tenth problem: Near Marmet, in the river Eraskh (Arax) I caught a fish. I scaled it. It simplified that it's head was the quarter of his weight, his tail was $\frac{1}{6}$ of his weight, his trunk was 140 liters. Now, figure out how much weight of the fish was.

Answer: The fish was 240 liters.

Eleventh problem: One merchant passed through three towns. In every town he gave the half and $\frac{1}{3}$ part of property as a duty. When he was at home, he had only 11 Dahekans. Now, you must figure out how many dahekans he had.

Answer: The merchant had 2376 Dahekans.

Twelfth problem: I wanted to make a ship, but I had only three Drams. I said to my relatives, "Let everyone of you give me something and I'll make a ship". One of them gave me the price of the $\frac{1}{3}$ part of the ship, another one gave the $\frac{1}{4}$ part, someone gave the $\frac{1}{6}$ part, another gave the $\frac{1}{7}$ and one of them gave me the price of the $\frac{1}{28}$ part of the ship. So I made the ship. Now, you must figure out, how many Dahekans the ship was.

Answer: The ship costs 42 Dahekans.

Thirteenth problem: One of my students bought apples from Kara and wanted to give to me. In the way he met three groups of jokers. Every group took the half and the quarter of apples. He gave me rest of apples [5 ones]. Now, you must figure out how many apples he had.

Answer: He had 320 apples.

Fourteenth problem: There was wine in a pitcher. There were also three jars. I filled the wine into the jars. One of the jars took $\frac{1}{3}$, another took $\frac{1}{6}$ and the last one took the $\frac{1}{14}$ of the wine. The remaining wine, which was filled into the bowls was 54 Pases. Now, you must figure out how many Pases was the whole wine.

Answer: The wine was 126 Pases.

Fifteenth problem: I had a thoroughbred horse. I sold it. With the quarter of the money I got from the horse I bought cows. With the $\frac{1}{7}$ part I

bought goats, with the $\frac{1}{10}$ part I bought oxen and with the rest 318 Dahekans I bought sheep. Now, you must figure out how many Dahekans I got from the horse.

Answer: The price of the horse was 616 Dahekans.

Sixteenth problem: I was building a church. I hired a mason, who was putting 140 bricks in a day. 39 days later I hired another mason, who was putting 218 bricks in a day. When the number of the bricks that the second mason put equaled the number of bricks that the first mason put, the work was finished. Now, you must figure out how many days after [I hired the second mason] the work finished.

Answer: The work finished 70 days after I hired the second mason.

Seventeenth problem: A ship full of wheat was sailing. A whale was pursuing it. Sailors were afraid of the whale and threw half of the wheat to it. The next day they threw him the $\frac{1}{5}$ of the remaining wheat. The third day they threw him the $\frac{1}{8}$ of the wheat. At the fourth day they threw $\frac{1}{7}$ of the rest. When they were at the port, they had 7200 [kayts] wheat. Now, you must figure out, how much wheat they had.

Answer: They had 24000 kayts wheat.

Eighteenth problem: I had a metallic tray, which I broke and made plates. From $\frac{1}{3}$ I made one, from the $\frac{1}{4}$ part I made another one, from the $\frac{1}{5}$ part I made two cups, from the $\frac{1}{6}$ part a tray and from 210 drams I made a jar. Now, you must figure out how much was the weight of metallic tray.

Answer: The metallic tray was 4200 drams.

Nineteenth problem: A man entered three churches. In every church he asked from the God to give him as much money as he had and promised to give 25 Dahekans. After that he had nothing. Now, you must figure out how much money he had.

Answer: He had $21 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ (21.875) Dahekans.

Twentieth problem: Hunting-place of the governor of Shirac and Arshakunik, Nerseh Kamsarakan, was in the base of the mountain Artin. One night a herd of deer entered there. Servants did not have hunting

experience. They ran to the village Talin and told about that. Kamsarakan with his brothers and noble men hurried to the hunting-place. The slaughter of deer began. Half of them was caught in trap, the quarter was hit by arrows, the young deer which were $\frac{1}{12}$ of the herd, were caught alive and 360 ones were killed by spear. Now, you must figure out, how many deer were in the herd.

Answer: There were 2160 deer in the herd.

Twenty-first problem: The son of Arshavir, Nerseh Kamsarakan, who was the namesake and ancestor of this Nerseh, wins the war against nomads and takes many captives. He presents half of the capties to the Persian king and $\frac{1}{7}$ of the another half he gives to his son. Then he returns to his lands. In the way he visits one of the ministers and gives him $\frac{1}{8}$ of the rest of prisoners. After that he visits Khoravaran and gives him the $\frac{1}{14}$ of the remaining prisoners. When he enters his lands his youngest brother, Hrahat, goes to meet him. He gives him $\frac{1}{13}$ of the remaining prisoners. Then Armenian noble men go to meet him and he gives them $\frac{1}{9}$ of the rest.. When he reaches to Vagharshapat he gives $\frac{1}{16}$ of the rest to churches. Finally he gives $\frac{1}{20}$ of the rest to his elder brother, Sahak. So he gets only 570 men. Now, you must figure out how many captives he took.

Answer: He took 2240 captives.

Twenty-second problem: The pharaoh of Egypt was celebrating his birthday. He had a habit to distribute one hundred pitchers of wine to his ten ministers by their merit and position. Now, you must distribute one hundred pitches of wine among ten ministers.

Answer: First minister gets $1 + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{55}$ pitcher of wine. Second minister gets $3 + \frac{1}{2} + \frac{1}{10} + \frac{1}{40} + \frac{1}{80}$ pitchers of wine. Third minister gets $5 + \frac{1}{3} + \frac{1}{15} + \frac{1}{44} + \frac{1}{60} + \frac{1}{66}$ pitchers of wine. Fourth minister gets $7 + \frac{1}{5} + \frac{1}{20} + \frac{1}{44}$ pitchers of wine. Fifth minister gets $9 + \frac{1}{11}$ pitchers of wine. Sixth minister gets $10 + \frac{1}{2} + \frac{1}{5} + \frac{1}{10} + \frac{1}{22} + \frac{1}{30} + \frac{1}{33}$ pitchers of wine. Seventh minister gets $12 + \frac{1}{2} + \frac{1}{10} + \frac{1}{22} + \frac{1}{30} + \frac{1}{33} + \frac{1}{55}$ pitchers of wine. Eighth minister gets $14 + \frac{1}{3} + \frac{1}{10} + \frac{1}{15} + \frac{1}{22}$ pitchers of wine. Ninth minister gets $16 + \frac{1}{5} + \frac{1}{10} + \frac{1}{22}$

+ $1/55$ pitchers of wine. Tenth minister gets $18 + 1/12 + 1/22 + 1/33 + 1/44$ pitchers of wine.

Twenty-third problem: There was a cellar in which were two hundred kayts of barley [1 kayt=414720 grains]. Mice ate all the barley. I caught one and punished. He shrived and said, "I ate eighty grains". Now, you must figure out how many grains of barley were in the cellar and what was the number of the mice that ate the barley.

Answer: There were 82,944,000 grains in the cellar and the number of the mice that ate the barley was 1,036,800.

Twenty-fourth problem: There was a basin in Athens, which had three faucets. The first faucet filled up the basin in an hour, second faucet filled up the basin in two hours and the third faucet filled up the basin in three hours. Now, you must figure out in what time three faucets could fill up the basin.

Answer: All faucets at once could fill up the basin in $1/4 + 1/6 + 1/12 + 1/22$ [0.54(54)] hour.

Appendix 2

Mathematical Entertainment of Anania Shirakatsi

“I am writing to you, math funs, while you are eating and having fun and you want to say a joke or a pleasant expression, so you can take advantage of them”.

First Entertainment:

Tell your friend: "I can find out when you want to dine and how many times you drink wine." If he says that: "Find it out". So tell him: "Keep in mind the number of hours you want to eat. Repeat it, add five to it. Multiply (Total) by five. Add ten to it and multiply by ten. Add number of times you want to drink wine. When he did what you told him, you can ask him what is the sum of the numbers.

Whatever number that says, subtract 350 from it, then look how many hundreds there are, at that time number of dining will be known and number of drinking wine is less than a hundred. But if your friend is not experienced and the number of drinks was hundred, tell him that one hundred drinks in one hour are impossible".

Second entertainment:

"Tell your friend that once a Persian traveler saw a group of Greek tourist in a feast, who shouted: " If you were given to me, again you were given, again half of you and one quarter of you, I would be a hundred with you". So tell me number of the Greek tourists. If your friend is a wise man, he will immediately say that the Greeks were 36 and if he is not wise, his searches and lack of his knowledge will make you laugh and have fun ".

Third entertainment:

"Tell your friend that I can say to you how many Dirhams are in his bag. If he says: "go and find it out", then tell him get all his money and add the same amount, then add the first number in addition to it, double the resulting amount". If he does all of these, whether the number is odd or even, divide it by ten. The resulting number will be money in the bag".

Fourth entertainment:

"Tell your friend that a Hun⁴⁸ farm worker has worked for me a hundred years and he has shipped 100 eggs per day. Now you should know the whole numbers. If your friend is smart on numbers, he can answer quickly 365 bivar⁴⁹ eggs are shipped and if he is stupid he will get into trouble and you can laugh at him".

Fifth entertainment:

"Tell your friend:" If you sell sixty pitcher of wine, the price of five pitchers is two Dirhams, how many Dirhams will you gain?." He will answer 24 Dirhams. Tell him that, "If I sell 60 pitchers of wine at the same price, I will profit one Dirham more than you. You need to do such an act: Divide 60 into two, divide every three pitchers of 30 by one Dirham and divide second 30 by two Dirhams, thus, still five pitchers are considered at two dirhams and one dirham more than the first benefit, and you will have fun from his being surprised".

⁴⁸ - The Huns were a nomadic group of people who are known to have lived in Eastern Europe, the Caucasus, and Central Asia between the 1st century AD and the 7th century. They were first reported living east of the Volga River, in an area that was part of Scythia at the time; the Huns' arrival is associated with the migration westward of a Scythian people, the Alans. They were first mentioned as Hunnoi by Tacitus. In 91 AD, the Huns were said to be living near the Caspian Sea and by about 150 had migrated southeast into the Caucasus. By 370, the Huns had established a vast, if short-lived, dominion in Europe.

⁴⁹ - Bivar = 10,000

Appendix 3

Polygonal numbers of Hovhannes Sarkavak⁵⁰

⁵⁰ -Manuscript No. 8973, at the National Library of manuscripts of Yerevan, Matenadaran, Page 79-84, written in Theodosia of Crimea in 1445.

[illegible]

[illegible]

[illegible]

[illegible]

Appendix 4

Examples of the mathematical problems by Nikoghayos (Nikolai) Artavazd

[Summary] of the letter to my dear friend Theodore from Nikoghayos Artavazd of Byzantium

Due to your interest and perseverance in the interpretation of problems related to the numbers, I try to present it in order. It consists of applications in four important fields, which are arithmetic, geometry, astronomy and music.

Examples: Someone asked another one about product of $3 + 1/3 + 1/14 + 1/42$ multiplied by itself? He did not have a solution to it. I want to tell you a method that I think a lot of people are unfamiliar with it, convert the fractions with a 42 denominator, ie one-third to $14/42$, $1/14$ to $3/42$ and the total will be $18/42$ which is equal to $3/7$, for better clarity I have left 42 and taken 7, then I convert 3 to $21/7$ which with $3/7$ totals to $24/7$. So 24 multiplied by itself equals 576. Also $1/7$ multiplied by itself gives $1/49$, then divide 576 by 49.

The result is 11 units and $37/49$ that equals $2/3 + 1/12 + 1/196$ units. Therefore $3 + 1/3 + 1/14 + 1/42$ multiplied by itself is equal to $11 + 2/3 + 1/12 + 1/196$.

The second example, multiplication of $5 + 2/3 + 1/5 + 1/33 + 1/110 + 1/330$ by $8 + 2/3 + 1/4 + 1/156$ is equal to $52 + 2/3 + 1/18 + 1/429 > 1/2374$.

A few examples of this kind:

1. A person who asks somebody that $\frac{1}{5}$ and $\frac{1}{6}$ of my money is equal to 21. How much is my total money? He, who was a trained person and knowledgeable responded briefly $57 \frac{3}{11}$.

Solution: multiply denominators ($30 = 5 \times 6$) and by 21 resulting 630. Add 5 and 6 (11), divide 630 by 11. The result is $57 \frac{3}{11}$.

2. Again he said, "I picked up the $\frac{1}{4}$ and $\frac{1}{5}$ of the money from the fund and the remaining money was 12. I want to know how much was the total money. Answer is $21 \frac{9}{11}$.

Solution:

$$4 \times 5 = 20$$

$$20 \times 12 = 240$$

$$4 + 5 = 9$$

$$20 - 9 = 11$$

$$240 / 11 = 21 \frac{9}{11}$$

3- And he said to them again I added $\frac{1}{4}$ and $\frac{1}{5}$ to my money and result was 30. What I want to know how much was my money before adding $\frac{1}{4}$ and $\frac{1}{5}$. Answer is $20 \frac{20}{29}$.

Now the solution:

$$4 \times 5 = 20$$

$$4 + 5 = 9$$

$$20 + 9 = 29$$

$$30 \times 9 = 270$$

$$270 / 29 = 9 \frac{9}{29}$$

$$30 - 9 \frac{9}{29} = 20 \frac{20}{29}$$

4. Someone asks another person I did a deal and purchased the amount $3\frac{1}{3}$ of goods sold it and purchased again at $3\frac{1}{5}$, again I sold it and noticed that I have earned 10 money. I want to know how much I invested. The second person answers 240.

Solution: $\frac{1}{3}$ and $\frac{1}{5}$ is got of 15:

$$15 \times 3\frac{1}{3} = 50$$

$$15 \times 3\frac{1}{5} = 48$$

$$48 \times 10 = 480$$

$$50 - 48 = 2$$

$$480 / 2 = 240$$

5. I gave 7 gold coin to my servant to buy green and blue fabric. But the prices of these two are not equal, price of blue fabric as per each sleeve is $1\frac{1}{2}$ coin and green one is coin $2\frac{1}{2}$ coin. I want to know how much you should pay for each.

$$1\frac{1}{2} + 2\frac{1}{2} = 4$$

$$(7+4) \times \frac{1}{4} = 1\frac{1}{2}\frac{1}{4}$$

$$(1\frac{1}{2} \times 7) + 4 = 2\frac{1}{2}\frac{1}{8}$$

$$(2\frac{1}{2} \times 7) / 4 = 4\frac{1}{3}\frac{1}{24}$$

So you should buy $1\frac{1}{2}\frac{1}{4}$ sleeves of each fabric and pay $2\frac{1}{2}\frac{1}{8}$ for blue and $4\frac{1}{3}\frac{1}{24}$ for green.

6. A person is willing to live again $\frac{1}{3}$ and $\frac{1}{5}$ of the years he has already lived. His will was done. When did he raise such a desire?

Answer: 90.

Solution:

$$15 = 5 \times 3$$

$$5 + 3 = 8$$

$$23 = 8 + 15$$

$$2070 = 15 \times 138$$

$$2070 \div 23 = 90$$

Appendix 5

A glossary of mathematical terms

In this section we present brief glossary of mathematical terms written by Sahak Prunian. It is included at the end of “Geometry” by Prunian, the 18th century mathematician⁵¹.

In the glossary mathematical terms are presented in Armenian, Latin/Italian, Arabic/Turkish/Persian. In order to get familiar with the mathematical terminology of the Armenians in the 18th century We present the Armenian term then its pronunciation then the English translation. Naturally, the terminology of mathematics in modern Armenian language has been developed a lot and some special dictionaries have been prepared, which We have used some of them while writing this book:

Armenian word – pronunciation - meaning

Աղեղն Agheghn bow

Աղեղնադիր անկիւն Agheghnadir angiun bowed angle

Անկիւն Angiun angle, corner

Անհասարակողմեան Anhavasarakoghmyan of parallelogram

Աշխարհագրութիւն Ashkharhagutiun Geography

Ապացուցութիւն Apatsutsutiun argument

⁵¹ -S. Prunian, “Geometry”, Venice 1794, pp. 415-420.

Առած Arrads like

Առանցք Arrantsk axis

Առումն Arrumn harvest, fraction

Աստիճան Astijan degree

Աստղաբաշխություն Astghabashkhutun աստրոնոմը

Արտակենտրոն Artakedron outside the center circle

Բազմանկյուն Bazmangiun polygonal

Բազմապատիկ Bazmapatik manifold

Բացահայեաց գիծ Batsahayats gids open line, non-line package

Բթանկիւն Btangiun obtuse angle, angle

Բոլորակ Bolorak circle

Բութանկիւն But-angiun acute angle

Բուրգ Burg pyramid

Գագաթն Gagatn top

Գիծ Gids line

Գիծ կոր Gids kor curved line, arc

Գիծ ուղիղ Gids ughigh straight line, right

Գլան Glan cylinder

Գծաչափ Gdsachap segment

Գնդական Gndakan spherical

Գնդաձև Gndadzev spherical

Գունդ Gund sphere

Գրահաշիւ Grahashiv algebra

Դրութիւն Drutyun hypothesis, thesis

Ենթադրութիւն Yentadrutyun hypothesis, proposition

Եռակողմ Yerrakoghm triangular, three-dimensional

Եռանկիւն Yerrankyun triangular, triangle

Եռանկիւնաչափութիւն Yerrankyunachaputyun trigonometry

Եռապատիկ Yerrapatik triple, triple, third degree

Եռապատկեալ Yerrapatkyal triple

Երաժշտութիւն Yerazheshtutyun music

Երկայնաչափութիւն Yerkeynachaputyun length measurement

Երկրաչափութիւն Yerkrachaputyun geometry

Եօթնանկիւն Yotnankyun seven corners, Heptagon

Չանգւած Zangvads mass

Չուգահեռագիծ Zugaherragids parallel lines

Չուգահեռական Zugaherragan parallel

Չուգահեռաստն Zugaherravotn parallelepiped

Ընդարձակություն Endardzagutyun level, space, right

Ընդդիմազագարն Enddimagagatn front head

Ընդհայեցողություն Endhayetsoghutyun view

Ընդօրինակություն Endorinakutyun view, perspective

Թուարանություն Tvabanutyun arithmetic

Ժամանակագրություն Zhamanakagrutian chronology

Լար Lar cord, chord

Լուծումն Ludsumn solution

Լրում /բովանդակված Lrumn / bovandakvads all, perfection

Խառնագիծ Kharnagids cross line

Խարիսխ Khariskh anchor, torque

Խնդիր Khendir question

Խոտորնակ Khotornak crooked, bent

Խոտորածիկիծ Khotoradzig- gids broken line

Խորանարդ Khoranard, cube, cubic

Խորովիք Khoropik of concave, concave

Խորություն Khorutyun depth

Կամար Kamar bow

Կանոնափակ Kanonapak focus, regular

Կեդրոն Kedron Center

Կենտրոնադիր անկիւն Kentonadir ankiun central angle

Կէտ Ket point

Կիսաբոլորակ Kisabolorak semicircle

Կիսագունտ Kisagunt Hemisphere

Կիսատրամագիծ Kisatramagids radius

Կողմն Koghmn Zl- way, side

Կոնոն Konon cone

Կորագիծ Koragids curved line, curved line

Կորնթիկ Korntik hump, convex

Կրկնապատկել Kerknapatkel double

Հակադիր Hakadir opponents, mutual

Հակուղիղ Hakughigh chord (right triangle)

Համադիր Hamadir peer, the corresponding

Համակենտրոն Hamakentron concentric

Համակողմեան Hamakoghman nearby, all-round

Համեմատականութիւն Hamematukanutiun proportion, ratio

Համեմատութիւն Hamematutiun ratio, comparison

Հայելաբանութիւն Hayelabanutyun of mirrors

Հաստատաչափություն Hastatachaputyun measurement

Հաստատուն Hastatun three-dimensional, solid

Հատանող Hatanogh decisive, Secant

Հատուած Hatvads piece, part, segment

Հատուածագիծ անկյուն Hatvadsagids ankyun inscribed angle

Հատուածակողմ Hatvadsakoghm charter

Հատուածող Hatvadsogh sector

Հաւասարակողմնան Havasarakoghmian equilateral

Հաւասարաբազմապատիկ Havasarabazmapatik the multiples, the coefficient

Հաւասարանկյուն Havasarankyun equilateral

Հաւասարախոտոր / Մակարդակ Havasarakhotor / makardak similar slope Mirrors

Հաւասարամասն / Գիծ Havasaramasn / gids front lines, line segments equal

Հետևանք Hetevank result, interest

Հետևորդ / Եզր Hetevord yezr, outcome

Հնգանկյուն Hngankyun pentagon

Ձև Dzev shape, configuration, table

Չողաչափ / Չող Dzoghachap / dzogh bar, tally

Ճարտարապետություն Chartarapetutian Architecture

Մակագծեալ Makagedsyal environment

Մակարդակ Makardak level

Մակարդակաչափություն/Մակերևութաչափություն/Makardakachaputyun /
Makerevutachaputyun plane geometry, measurement levels

Մակերևոյթ Makerevuyt level, surface

Մանրամասն Manramasn close, detailed

Մանրերկրորդ Manrerkrord seconds

Մատնաչափ / Մատն Matnachap / matn finger

Մարմին Marmin body

Մեծահայաց գիծ Merdzahayats gids line converge, convergent

Մեքենականգիտություն Mekenakan gidutiun mechanics, the science of
art

Միջակետ Mijaket center

Միջոց Mijots level, space, right, the

Յառաջատություն Harajadutyun progression

Յարանուանող Haranvanogh denominator

Նախադասություն Nakhadasutyun including, introduction, sentence

Նախադասություն Գործնական Nakhadasutiun gordsnakan issue

Նախընթաց Nakhentats previous

Ներքնախարիսի Nerknakhariskh internal torque

Նոյնակեդրոն Nuynakedron concentric

Շառավիղ Sharravigh radius

Շարունակ Sharunak connected, continuous

Շեղանկին / Խոտորանկին Sheghankyun / khotorankyun diamond

Շրջանակադիր անկին Sherjanakadir ankiun right angle, the right corner

Շրջապատ Sherjapat circle, environment

Շրջափակ Sherjapak surrounding

Շոշափող Shoshapogh tangent, tangent

Ոտնաչափ/ Ոտն Votnachp (votn) step

Որչափական Vorchapakan part, quantity

Ուղղագիծ Ugghagids straight line

Ուղղահայած գիծ Ugghahayats gids vertical line

Ուղղանկին Ugghankyun right angle, the right corner

Ուսումնականութիւն Usumnakanutyun Mathematics

Պարագցեալ Paragedsial environment

Պարադրեալ Paradrial inscribed

Պարականոն Parakaron anomaly

Պարաչափ Parachap environment

Ջրաբաշխութիւն Jrabashkhutium hydraulic science

Ջրագիտութիւն Jragidutium hydraulic science

Սահման Sahman defined border

Ստորաբազմապատիկ Storabazmapatik joint scaling split line, divisor

Ստորագրեալ Storagrail inscribed

Ստուերաչափութիւն Stverachaputyun under assessment

Սրանկիւն Srankiun an acute angle, sharp angles

Սուրանկիւն Surankiun acute angle, sharp angles

Վեցանկիւն Vetsankiun hexagon

Տարանկիւն Tarankiun diamond

Տարանկիւնային Tarankiunayin diamond

Տարանջատ / Տարոռշ Taranjat / tarorosh discrete, disconnected, isolated

Տեսաբանութիւն Tesabanutium of light, optics

Տրամագիծ Tramagids diameter

Տրամանկիւն Tramankiun (angle) diagonal

Փոխադարձ Pokhadardz reverse, cross

Փոխադարձութիւն Pokhadardzutium conversion and delivery

Փոփոխութիւն Popokhutium conversion, handling

Քանակութիւն Kanakutium quantity

Քանոն Kanon law

Քանորդական Kanordakan part, portion

Քառակողմ Karakoghm Quadrilateral

Քառակուսի Karakusi square

Քառանկիւն Karankiun four corners

Քառապատկեալ Karapatkial four times, the fourth

Քառորդ գործի Karord gordsi one-quarter

Օդաչափութիւն Odachaputyun climate science, meteorology

Index

- 6-hazariak, 56
A. A. Talalyan, 169
A. B. Nersesyan, 169
A. H. Hovhannisyan,, 169
A. Shahinian, 166, 167, 168
Abdulhamid, 24
Academy of Sciences of Armenia,
 168, 171, 173, 174, 183, 185, 188
Achaemenids, 40
Achemenide, 16
acrophonic, 46
Addition, 38, 61, 62, 67
Aghavard Khachatryan, 186
Aghtamar, 93
Akkadian, 16, 33
Albania, 17
Albert Petrosyan, 188
Alexander (Sumbat) Abian, 172
Alexander Hacopian, 166
Alexandr Talalyan, 188
Alexandria, 10, 60, 87
Algebra, 12, 186
Al-Khwarizmi, 10
Alp Arslan, 93
Alphabet, 45
Anahit Chubaryan, 183
Anahit Galstyan, 183
Anahit Harutyunyan, 185
Anania Shirakatsi, 50, 52, 53, 55,
 59, 61, 62, 67, 69, 73, 78, 79, 80,
 104, 107, 174, 193, 199, 226
Anatolia, 14, 23, 25
Anatoly Kitbalyan, 186
Ani, 20, 52, 84, 93, 94, 95, 116
Anry Nersesyan, 187
Antonius Portoli, 153
Araik Poghosyan, 188
Arakel Taslakyian, 189
Aram Hakobyan, 184
Aram Khachatryan, 186
Ararat, 16, 28, 40
Araratian, 27, 28
Archesh, 84, 93
Ardske, 93
Ardsn, 84, 93
Argishti, 16
Arman Karapetyan, 186
Armavir, 16, 40
Armen Jerbashyan, 185
Armen Kamalyan, 185
Armenian Genocide, 24, 176
Arnak Poghosyan, 188
Arpi Stepanyan, 188
Arsacid, 41
Arsacids, 18
Arshak Tonian, 166, 167
Arshakuni, 18, 19
Arshakunian, 18
Arshamashad, 40
Artashes I, 16, 40
Artashesian dynasty, 16, 40, 139
Artavazd, 107, 108, 109, 110, 111,
 113, 114, 115, 206
Artur A. Sahakian, 169
Artur Barkhudaryan, 183

- Artur Hovhannisyan, 185
Artur Sahakyan, 188
Ashkharhatsuyts, 58
Ashot Abrahamian, 53, 61, 88
Ashot Galumyan, 183
Ashot III, 84
Ashot Megerdichian, 166
Ashot Petrosyan, 179
Asia Minor, 17, 18, 116
Assyria, 15, 36
Astrakhan, 137
Athenic, 45
Athens, 46, 60, 198
atibi, 35
Atropatene, 17
Attic numerals, 46
Avarayr, 19
Avesta, 26
Avetik Arakelyan, 182
Ayrarat, 28
B. Farahvashi, 26
B. T. Batikyan, 169
Babylonian, 9, 14, 32, 33, 34, 35
Bacon, 95
Bagrat Batikyan, 189
Bagratuni, 20, 21, 84, 85, 93
Bagratunian dynasty, 20
Bahadur Bhadrian, 166
Bats-droutyoun, 66
Bazmapatik, 68, 210
Bejni, 85
Bel, 14
Bernoulli, 12
Biruni, 10
bivar, 48, 200
Borik Sahakyan, 188
Byurakan Astrophysical
Observatory, 53
Byzantine, 19, 20, 45, 50, 52, 55,
60, 61, 82, 84, 85, 107, 114, 115
Byzantines, 21, 22
Calculation system, 30
Calendarology, 56
Canton, 36
Catholicos, 19, 56, 92
Caucasus, 25, 28, 200
Chaldeans, 45
China, 40, 51
Christian, 19, 22, 54, 114
Cilicia, 21, 22, 24, 83, 115
Constandin, 82
Constantinople, 20, 23, 24, 45, 52,
81, 82, 115, 116
Counting system, 27, 45
Crusades, 22
cybernetics, 13
Cyprus, 22
D. Hilbert, 12
d'Alembert, 12
Dark, 73, 74
Davit Anhaght, 44, 45
Der-el-Zor, 25
Diaspora, 25, 134, 172
divisions, 73, 74, 127
Dmitry Mirimanoff, 181
Dsots, 162
Dvin, 19, 84, 93, 116, 193
Eastern Armenia, 22, 23

- Ed. Germanic, 40, 43, 45, 226
Edie Baghdasarian, 1, 8, 40, 43, 45, 85, 226
Egyptian, 9, 22, 36, 45, 112
Egyptians, 22, 45, 56, 59, 79
endouneloutyoun, 62
Ernst Nazaryan, 187
Euclid, 10, 11, 59, 82, 84, 86, 87, 88, 90, 92, 141, 142
Euler, 12
February, 12, 177, 179
Ferdowsi, 26
French, 22, 108, 120, 122, 139, 152, 181, 226
G. Gevorkyan, 169
Gadmokh, 15
Gagik Hakobyan, 184
Gagik Mikaelyan, 186
Galra, 126
Garegin Petrosyan, 173
Garni, 41
Gauss, 12
Gayane Ghazaryan, 184
Gaz, 137
Gegham Gevorkyan, 183
Geometry, 88, 159, 160, 161, 164, 175, 186, 209
Georg, 108, 109
George Piranian, 180
George Sahakyan, 188
Georgia, 17, 25, 93, 116, 171, 181
Georgians, 56
Ghazaryan, 171, 184
Gohar Harutyunyan, 185
Grabar, 47
Great Armenian Encyclopedia, 26
Greco-Roman polytheism, 18
Greek, 10, 15, 19, 40, 43, 46, 47, 48, 59, 61, 85, 86, 87, 90, 91, 104, 107, 108, 109, 114, 117, 139, 143, 199
Grigor Barsegyan, 183
Grigor Kesaratsi, 143
Grigor Magistros, 88, 92
Grigori Karagulyan, 186
Gro Hovhannisyan, 185
Gurgen Asatryan, 182
Gurgen Hakobyan, 184
Guy Terjanian, 181
H. Badalian, 167
Haghat, 93, 94, 95
Hakop Hakopyan, 185
Hakop Meghapart, 134
Hamadsots, 162
Hatanogh, 162, 214
Hatoghakits, 162
Hayk, 14, 40, 171, 184, 186
Heptagon, 100, 211
Hexagonal, 100
Hopital, 12
Hovhannes Navakatikian, 166, 167
Hovhannes Sarkavak, 93, 94, 95, 97, 101, 104, 106, 107, 201
Hovsep Orbeli, 31, 61
Hrachia Atcharian, 108
Hrachik Hairapetyan, 184

- Institute of Mathematics, 169, 170,
179, 180, 182, 183, 184, 185,
186, 187, 188, 189
Ishkhan Khachatryan, 189
Jews, 56
John Napier, 11
Kalim Stephanos, 133
Kamani, 35
Karabakh, 26
Karen Avetisyan, 182
Karen Navasardyan, 187
Karen Yagdjian, 189
Kars, 84, 93
Kassites, 33
Kepler, 11
Kh. Torgomian, 166
Khachatur, 108, 157, 186
Khazars, 52
Khorenatsi, 26, 29, 44
Khromov, 171
Knnikon, 53, 56
koppa, 46
Kotchatk, 73, 74
Kura, 17
Kurdish, 25, 93
Kvayk, 100
Lagrange, 12
Laplace, 12
Lavrenty Matevosyan, 186
Legendre, 12
Leningrad, 36, 166
Leon, 81, 82, 83, 115
Leonid Khachiyan, 176
Levon Arabajyan, 182
Levon Gevorgyan, 183
Levon Mikayelyan, 187
Liparit Tepoyan, 189
Lori, 93
M. Jerbashian, 167, 168
M. S. Ginovyan, 169
M. Toumanian, 8
Macedonians, 56
Magistros, 82, 84, 85, 86, 88, 92,
104, 107
Maksim Muradyan, 187
Mamikon A. Mnatsakanyan, 178
Mamikon Ginovyan, 184
Manazgerd, 93
Martin Grigorian, 184
massacres, 24
Matenadaran, 7, 63, 66, 67, 69, 73,
88, 89, 96, 97, 99, 118, 121, 122,
127, 133, 136, 141, 153, 201, 227
Mathematical entertainment, 80
Mediterranean, 17, 21, 116
Megerdich Toomanian, 173
Menua, 29
Meshini, 35
Mesopotamia, 17, 33, 85
Mesrop Mashtots, 7, 43, 48, 53, 63,
66
Methghal, 137
Mher Martirosyan, 186
Michael Poghosyan, 188
Michaelis Clycus, 83
Middle Ages, 53, 82, 95, 107, 174,
226
Mkhitar Jerbashyan, 175

- Mkhitaryan, 141
Moks, 93
Mongols, 22, 114, 116
Moses Khorenatsi, 29
Multiplication, 61, 68, 69, 70
multiplier, 100, 125, 126
N. E. Tovmasian, 169
N. Gasparian, 166
N. Petrosian, 166, 167
N. U. Arakelyan, 169
Nagorno-Karabakh, 22
Naira Mesropyan, 186
Nairi, 14, 179
Nakhijevan, 78, 193
Nasir al-Din Tusi, 10
National Library, 7, 108, 109, 133, 136, 141, 153, 201, 227
Nazaret Tovmasyan, 190
Newton, 11, 12
Nicomachus, 59, 60, 80, 104
Nikoghayos, 107, 108, 109, 112, 206
Nikoghayos (Nokolay) Artavazd, 107
Nikoghayos Artavazd, 107, 108, 109, 112, 206
Norair Arakelyan, 182
Nune Hovhannisyan, 185
Omar Khayyam, 10
Ottoman, 22, 23, 24, 25, 117, 134
Parisos, 94
Parsavagirk, 82
Patchen Tomari, 56
pentagon, 100, 214
Persia, 16, 17, 18, 19, 22, 23
Persian, 8, 9, 16, 22, 48, 50, 78, 139, 143, 152, 173, 193, 194, 197, 199, 209, 226
Persians, 18, 19, 56, 77, 78, 193, 194
Philosopher, 82, 95
Quadrilatera, 101
R. V. Ambartzumian, 169
Rafayel Barkhudaryan, 183
Rafik Aramyan, 182
Raqam, 120
Red Army, 26
Republic of Armenia, 7, 16, 25, 175, 182
Rom Varshamov, 180
Roman, 18, 19, 46, 47, 60, 118, 194
Romans, 27, 56, 120
Rome, 17
Romen Shahbaghyan, 189
Rouben V. Ambartzumian, 175
Rshtuni, 93
Ruben, 21
Russian, 22, 23, 25, 26, 137, 140, 169, 178, 182, 226
S. Abajian, 166
S. Bahrani, 26
S. Mergelian, 167, 168
Sahak Narimanyan, 187
Sahak Prunian, 88, 159, 160, 162, 209
Sahak Prunyan, 164
sampi, 46
Sarduri, 31, 38

- Sargis Aleksanyan, 182
Sargis Hakobyan, 189
Sarkavak, 94, 95, 106
Sarmayeh, 152
Sassanid, 18, 19, 45
Sassoon, 93
Sasun, 22
Sergey Afyan, 182
Sergo Episkoposyan, 183
Shadadians, 93
Shirakatsi, 50, 52, 53, 54, 55, 56,
57, 58, 59, 60, 61, 62, 65, 67, 68,
71, 72, 73, 74, 75, 77, 78, 80, 92,
104, 115
Shoshapogh, 162, 216
Shoshapoghakits, 162
Shusini, 35
Silva Andriyan, 182
Six-thousands, 61, 72, 73, 74
Smbat Gogyan, 184
St. Denis Cathedral of Paris, 22
State University, 165, 168, 169, 172,
174, 177, 178, 179, 182, 183,
184, 185, 186, 187, 188, 189
Stella Papyan, 187
Stella Safaryan, 189
Stepanos Boghosian, 137
subtraction, 38, 47, 48, 56, 61, 66,
67, 68, 111, 123, 125, 131, 149,
154, 156, 163
Sukias, 154
Surmelian, 157, 158
Syria, 83, 116
Syrians, 17, 56
Syunik, 93
Taylor, 12
Tehran, 8, 26, 29, 40, 41, 43, 45, 85,
104, 226
Tekor, 41, 42
Theodore Rshtuni, 19
Tiglath Pileser, 15
Tigran, 17, 40, 185
Tigranagert, 139
Transcaucasian federation, 25
Triangular, 98, 101, 104
Trigonometry, 162, 164
Turkish, 22, 23, 25, 139, 152, 158,
159, 209
Turks, 22, 24, 25, 60, 93, 114
Tyukhik, 52, 55
Tyukhik's school, 55
United States., 25
Urartu, 9, 16, 17, 27, 28, 29, 30, 34,
35, 36, 38, 39, 40, 45, 48, 49, 226
V. A. Martirosian, 169
V. Ambartzumian, 170, 175
V. Saghatelian, 166
Vagarshak Ohanyan, 187
Vahan Sarafyan, 188
Vahram Dumanyan, 183
Van, 16, 27, 28, 31, 84, 93, 139, 176
Varazdat Navoyan, 187
Vardan Tepoyan, 189
Vartan Mamikonian, 19
Vartapet Aghamaliants., 154
Vazgen Avanesian, 173
Venetians, 22
Vertical, 100

- Vets-hazaryak, 72
Victor Ohanyan, 187
Vidok Musoyan, 189
Vigodsky, 36
Vrezh Amirkhanian, 173
Western Armenia., 22
Yereruk, 41
Yerevan, 7, 16, 26, 29, 33, 44, 51,
53, 63, 65, 66, 83, 86, 87, 88, 94,
95, 108, 109, 118, 133, 141, 153,
165, 166, 168, 169, 174, 175,
176, 177, 178, 179, 180, 182,
183, 184, 185, 186, 187, 188,
189, 201, 226, 227
Yervand Mkrtchyan, 187
Yervandakerd, 40
Yervandashad, 40
Yeznic Koghbatsi, 44
Yuri Hakopyan, 185
Zakarian, 93, 116, 169
Zbet Khojantian, 166
Zeytun, 22
Zorak Kamsarakan, 78, 193, 194
Zoroastrianism, 18, 19
Zvartnots, 50, 51

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